8 Appendix

In this Appendix, we first prove Proposition 2. We then test the null hypothesis that the countercyclical pattern of the relative capital productivity we obtained in Section 5.1 is purely driven by the demand channel.

8.1 Proof of Proposition 2

At period 1, when the news arrives, the current capital allocation follows

$$k_1^c = \left(\phi\beta V\left(K_2; Z^{new}\right)\right)^{1/\alpha},$$

and the current household wealth is

$$f_1 = (1-\delta) K_1 + \alpha Z^{old} \left(\frac{K_1 - \eta k_1^c}{1-\eta}\right)^{\alpha-1} K_1.$$

Suppose that $K_2 > K_1$ (this will be checked below). Then, the property $V_K(K; Z) > 0$ and $V(K; Z^{new}) > V\left(K; Z^{old}\right)$ imply that $V(K_2; Z^{new}) > V(K_1; Z^{new}) > V\left(K_1; Z^{old}\right)$, which gives

$$k_1^c = \left(\phi\beta V\left(K_2; Z^{new}\right)\right)^{1/\alpha} > \left(\phi\beta V\left(K_1; Z^{old}\right)\right)^{1/\alpha} = k_0^c.$$

Intuitively, the good news improves efficiency by allocating more capital to the type-c projects. Therefore, according to (6), aggregate TFP and output increase.

The household Euler equation gives

$$\frac{f\left(K_2; Z^{new}\right) - \Gamma\left(K_2; Z^{new}\right)}{f_1 - K_2} = \beta \frac{f\left(K_2; Z^{new}\right)}{K_2}.$$

Since $\Gamma\left(K_2; Z^{new}\right) = \beta f\left(K_2; Z^{new}\right)$, by (8), the above equation yields

$$K_2 = \beta f_1. \tag{37}$$

With $k_1^c > k_0^c$, it is clear that $f_1 > f_0 \equiv (1-\delta) K_1 + \alpha Z^{old} \left(\frac{K_1 - \eta k_0^c}{1-\eta}\right)^{\alpha-1} K_1$. The fact that $f_1 > f_0$, together with (37), confirms that $K_2 > \beta f_0 = K_1$. The period-1 household consumption and aggregate investment equal $f_1 - K_2$ and $K_2 - (1-\delta) K_1$, respectively. Since $f_1 - K_2 = (1-\beta) f_1 > (1-\beta) f_0 = f_0 - K_1$ and $I_1 = K_2 - (1-\delta) K_1 > K_1 - (1-\delta) K_0 = I_0$ (note that $K_1 = K_0$), both household consumption and aggregate investment increase on impact in response to the good news.

Finally, aggregate consumption increases if and only if $\frac{\partial Y}{\partial k^c} > \frac{\partial I}{\partial k^c}$ at the steady state. Note that

$$\begin{aligned} \frac{\partial Y}{\partial k^c} &= \frac{\partial TFP}{\partial k^c} F(K) \\ &= \eta \left(1/\beta - 1 + \delta\right) \left[\left(\frac{\phi\beta (1-\alpha) Z}{1-\phi\beta} \right)^{\frac{\alpha-1}{\alpha}} - 1 \right], \end{aligned}$$

where the third equality is obtained from the fact that at steady state, $ZF'(k^u) = 1/\beta - 1 + \delta$ and $\frac{F'(k^c)}{F'(k^u)} = \left(\frac{\phi\beta(1-\alpha)Z}{1-\phi\beta} \right)^{\frac{\alpha-1}{\alpha}}$. Moreover, we have

$$\begin{aligned} \frac{\partial I}{\partial k^c} &= \frac{\partial K'}{\partial k^c} \\ &= \frac{\beta\eta}{1-\eta} (1-\alpha) ZF'(k_1^u) \frac{K}{k^u} \\ &= \beta\eta (1-\alpha) (1/\beta - 1 + \delta) \left[1 + \frac{\eta}{1-\eta} \frac{k^c}{k^u} \right]. \end{aligned}$$

Therefore, $\frac{\partial Y}{\partial k^c} > \frac{\partial I}{\partial k^c}$ leads to the inequality (9).

8.2 Cyclical Financial Frictions Versus Cyclical Demand

Our finding of the cyclical pattern of the KP ratio in Section 6.1 can potentially be driven by the fact that small/young firms face more volatile demand fluctuations. To address this concern, we check the cyclical pattern of the KP ratio across industries with different levels of external finance dependence. The idea is that if the KP ratio between small/young and large/old firms is indeed driven by the demand channel, we should observe the same cyclicality across industries. Specifically, we first classify industries into two groups based on the degree of external finance dependence ("EFD" hereafter) measured by Rajan and Zingales (1998).[37] If an industry has an EFD above (below) the median, we categorize it into group $H(L)$ with high (low) EFD. We then categorize

[37] See Appendix 9.6 for details of constructing the measure of external finance dependence.

our sample firms into these two groups based on the industry they belong to. For each group $j \in \{H, L\}$, we run the following regression to estimate the KP Ratio between constrained and unconstrained firms.

$$\log KP_{it}^{j} = a_{t}^{j} + b_{t}^{j} d_{it}^{j} + \varepsilon_{it}^{j}, \quad j = H \text{ or } L.$$

Again, to control for the industry fixed effects on the measured capital productivity gap between the two types of firms, we add industry dummies at the 2-digit SIC level to the above equation.

The null hypothesis is that the countercyclical pattern of the KP Ratio is purely driven by the demand channel. If the hypothesis is true, we should expect that the correlation coefficient between b_t^H and GDP is not statistically different from its counterpart between b_t^L and GDP.

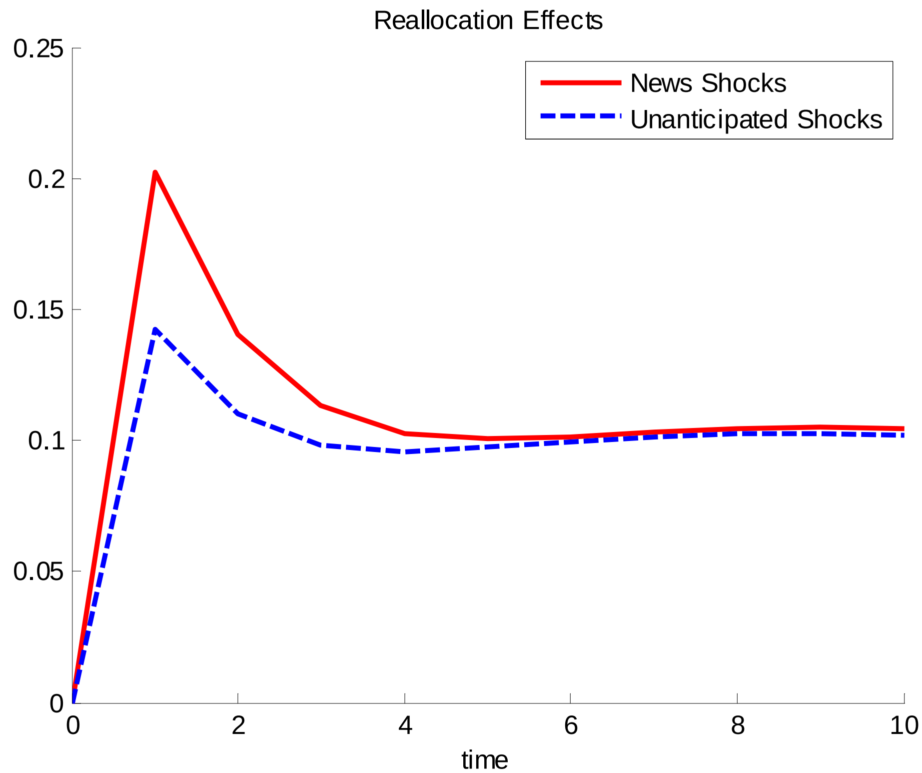
Table A.2 reports the results, with the middle column labeling the groups. We find that under both classification schemes, the KP Ratio for the group of high EFD are significantly more countercyclical than its counterpart for the group of low EFD. So, we can reject the null hypothesis that the countercyclical pattern of the KP ratio is purely driven by the demand channel.

Table A.2. Correlation of the Estimated KP Ratio with GDP and External Finance Dependence

Schemes	Group	Correlation with GDP
SA Index	High	−0.636
		(0.0000)
	Low	−0.495
		(0.0021)
Firm Size	High	−0.591
		(0.0000)
	Low	−0.373
		(0.0253)

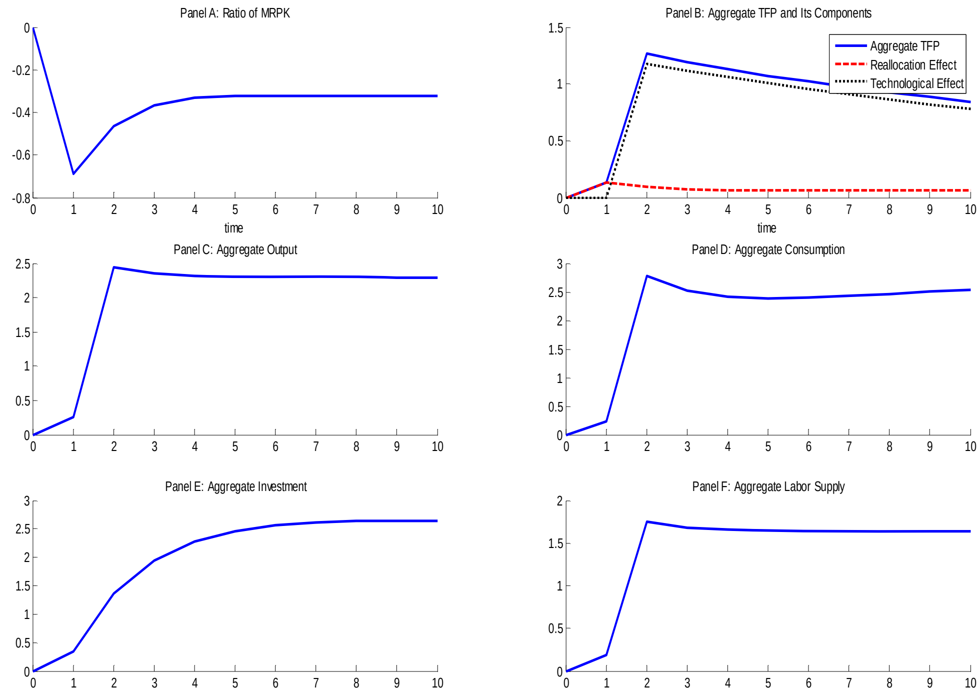
Note: This table presents, for two groups of firms differentiated by their industry EFD, the correlation coefficients between GDP and estimated relative productivity of constrained to constrained firms, both H-P filtered. A firm belongs to the subsample labeled "High" ("Low") if it belongs to an industry with high (low) external finance dependence. SA index and firm size refer to sorting firms by the SA index and one-year lagged book assets, respectively. The numbers in the parentheses are the p-values for testing the hypothesis of no correlation.

Figure A.1. Impulse Responses of Reallocation Effects to News and Unanticipated Shocks



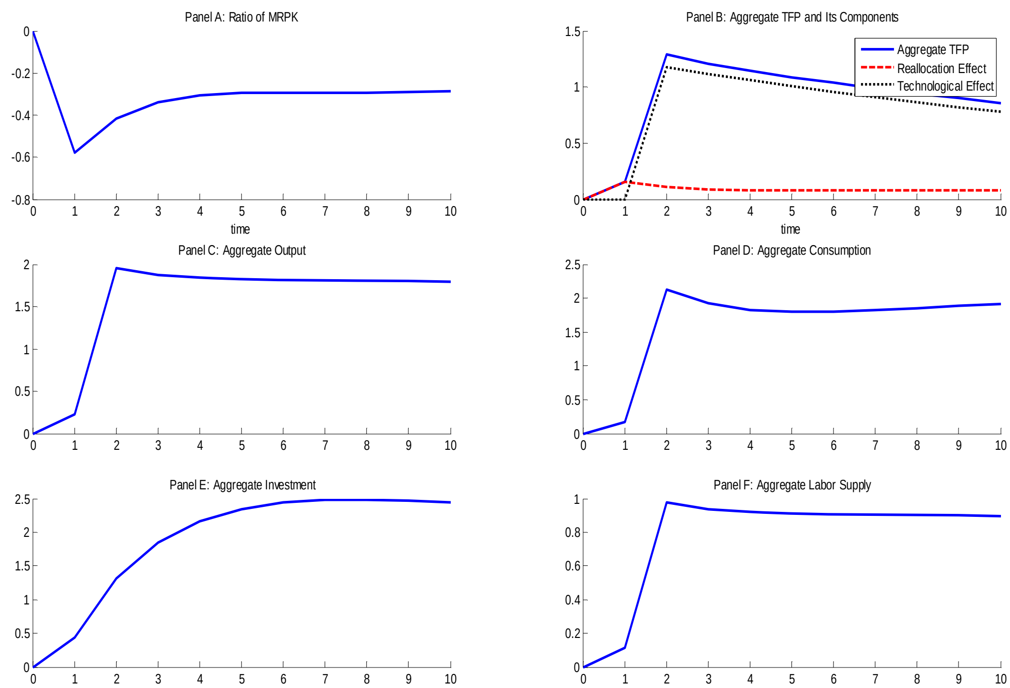
Note: The vertical axes denote percentage deviation from steady state.

Figure A.2. Impulse Responses to News Shocks on Aggregate Technology with η=0.132.



Note: The vertical axes denote percentage deviation from steady state.

Figure A.3. Impulse Responses to News Shocks on Aggregate Technology under ν=1.



Note: The vertical axes denote percentage deviation from steady state.