Appendix A – Proofs

Proof of Proposition 1

By contradiction. Suppose $\rho_2 > 0$. If $\mu_j > 0$, then $R_j = 0$ so equation (4) implies $i_L = \frac{\overline{\theta}X}{\psi}$. Substituting into equation (9) then implies $\xi_j > 0$ if and only if $\phi < \frac{(1+i_A)^2-1}{X} - \frac{\overline{\theta}^2 X}{\psi X} \equiv \overline{\phi}_1$ (where we have used $X_j = X$ in a symmetric equilibrium). If instead $\mu_j = 0$, then equation (7) implies $i_L = (1+i_A)^2 - 1$. Substituting into equation (9) then implies $\xi_j > 0$ if and only if $\phi < \frac{1-\overline{\theta}}{X}\left[(1+i_A)^2 - 1\right] \equiv \overline{\phi}_2$. Defining $\overline{\phi} \equiv \min\left\{\overline{\phi}_1, \overline{\phi}_2\right\}$ completes the proof. ■

Proof of Proposition 2

With $\rho_2 = 0$, the equilibrium is characterized by (4), (7), and:

$$\xi_j = \frac{\alpha\mu_j}{2\left(1-\overline{\theta}\right)}$$

$$\mu_j\left[R_j - \alpha\left(X - \omega\xi_j\right)\right] = 0 \text{ with complementary slackness}$$

There is an implicit refinement here since we are writing $\xi_j = \frac{\alpha\mu_j}{2\left(1-\overline{\theta}\right)}$ instead of $\xi_j = \frac{\alpha\mu_j\tau_j}{2\left(1-\overline{\theta}\right)}$. Both produce $\xi_j = 0$ if $\alpha\mu_j = 0$ so the refinement only applies if $\alpha\mu_j > 0$. Return to equations (8) and (9) with $\rho_2 = 0$ and $\alpha\mu_j > 0$. If $\xi_j > 0$, then $\eta_j^1 > 0$. This implies $\tau_j = 1$ which confirms $\xi_j > 0$. If $\xi_j = 0$, then $\eta_j^1 = \eta_j^0$. This implies $\tau_j \in [0,1]$. However, any $\tau_j \in (0,1]$ would return $\xi_j > 0$, violating $\xi_j = 0$. We thus eliminate $\xi_j = 0$ by refinement. Instead, $\alpha\mu_j > 0$ is associated with $\xi_j > 0$ and thus $\tau_j = 1$. For this reason, we write $\xi_j = \frac{\alpha\mu_j}{2\left(1-\overline{\theta}\right)}$. We can now proceed with the rest of the proof. There are two cases:

1. If $\mu_j = 0$, then $\xi_j = 0$ and $1 + i_L = (1+i_A)^2$. Equation (4) then pins down R_j. To ensure that $R_j \geq \alpha\left(X - \omega\xi_j\right)$ is satisfied, we need $\alpha \leq \overline{\theta} - \frac{\psi\left[(1+i_A)^2-1\right]}{X} \equiv \widetilde{\alpha}$. We have now established $\xi_j = 0$ if $\alpha \leq \widetilde{\alpha}$.

2. If $\mu_j > 0$, then complementary slackness implies $R_j = \alpha\left(X - \omega\xi_j\right)$. Combining with the other equilibrium conditions, we find that $\mu_j > 0$ delivers:

$$i_L = \frac{\alpha^2\omega\left[(1+i_A)^2 - 1\right] - 2\left(1-\overline{\theta}\right)\left(\alpha - \overline{\theta}\right)X}{\alpha^2\omega + 2\psi\left(1-\overline{\theta}\right)} \tag{21}$$

   Verifying $\mu_j > 0$ is equivalent to verifying $1 + i_L < (1+i_A)^2$. This reduces to $\alpha > \widetilde{\alpha}$. If $\widetilde{\alpha} \geq 0$, then we have established $\xi_j > 0$ with $\tau_j = 1$ for any $\alpha > \widetilde{\alpha}$.

Defining $\overline{\alpha} = \max\{\widetilde{\alpha}, 0\}$ completes the proof. ■

Proof of Proposition 3

Consider $\alpha = 0$. If $\mu_j = 0$, then (7) implies $i_L = (1+i_A)^2 - 1$ which is the highest feasible interbank rate. If instead $\mu_j > 0$, then the liquidity rule binds. In particular, $R_j = \alpha(X_j - \tau_j W_j)$ which is just $R_j = 0$ when $\alpha = 0$. We can then conclude $i_L = \frac{\overline{\theta}X}{\psi}$ from (4). Note that $\mu_j > 0$ is verified if and only if $\frac{\overline{\theta}X}{\psi} < (1+i_A)^2 - 1$.

Based on the results so far, we can see that the interbank rate at $\alpha = 0$ is independent of ρ_2. Let i_{L0} denote the interbank rate at $\alpha = 0$ and let $i_{L1}(\rho_2)$ denote the interbank rate at some $\alpha > 0$. From (4), we know $i_{L1}(\rho_2) = \frac{\overline{\theta}X}{\psi} - \frac{R_{j1}(\rho_2)}{\psi}$, where $R_{j1}(\rho_2)$ is reserve holdings at the $\alpha > 0$ being considered. The rest of the proof proceeds by contradiction. In particular, suppose $i_{L1}(\rho_2) > i_{L0}$. Then $\alpha = 0$ must be associated with $\mu_j > 0$, otherwise i_{L0} would be the highest feasible interbank rate and the supposition would be incorrect. We can thus write $i_{L0} = \frac{\overline{\theta}X}{\psi}$ and $i_{L1}(\rho_2) = i_{L0} - \frac{R_{j1}(\rho_2)}{\psi}$. The only way to get $i_{L1}(\rho_2) > i_{L0}$ is then $R_{j1}(\rho_2) < 0$ which is impossible. ■

Proof of Proposition 4

Start with general α. The derivatives of the big bank's objective function are:

$$\frac{\partial \Upsilon_k}{\partial \xi_k} \propto -\frac{2\omega\left(1-\overline{\theta}\right)}{1-\pi}\xi_k - \left[\frac{(1+i_A)^2 - 1}{1-\pi} - i_L^h\right]\frac{\partial R_k}{\partial \xi_k} + \left[\frac{(1+i_A)^2 - 1 - \phi X_k}{1-\pi} - \theta_h i_L^h\right]\frac{\partial X_k}{\partial \xi_k}$$

$$\frac{\partial \Upsilon_k}{\partial i_L^h} \propto R_k - \theta_h X_k - \left[\frac{(1+i_A)^2 - 1}{1-\pi} - i_L^h\right]\frac{\partial R_k}{\partial i_L^h} + \left[\frac{(1+i_A)^2 - 1 - \phi X_k}{1-\pi} - \theta_h i_L^h\right]\frac{\partial X_k}{\partial i_L^h}$$

It will be convenient to reduce these derivatives to a core set of variables (ξ_j, ξ_k, and i_L^h). If $\mu_j > 0$, then the complementary slackness in equation (13) implies:

$$R_j = \alpha\left(X_j - \omega\xi_j\right) \tag{22}$$

With $\delta_1 + \delta_2 = 0$ and $\overline{\xi_j} = \xi_j$, equations (10) and (11) are:

$$X_j = 1 - \delta_0 + \delta_1\left(\xi_j - \xi_k\right) \tag{23}$$

$$X_k = \delta_0 + \delta_1\left(\xi_k - \xi_j\right) \tag{24}$$

Substitute (22) to (24) into equation (12) to write:

$$R_k = \delta_0\theta_h + (1-\delta_0)\left(\overline{\theta}-\alpha\right) + \delta_1\left(\theta_h - \overline{\theta} + \alpha\right)\left(\xi_k - \xi_j\right) + \alpha\omega\xi_j - \psi i_L^h \tag{25}$$

Finally, combine equations (14) and (15) to get:

$$\xi_j = \frac{\alpha(1-\pi)}{2\left(1-\overline{\theta}\right)}\left[\frac{(1+i_A)^2-1}{1-\pi} - i_L^h\right] \tag{26}$$

We can now write $\frac{\partial \Upsilon_k}{\partial \xi_k} = 0$ as:

$$\xi_k = \frac{\delta_1\left[\left(1-\theta_h+\overline{\theta}-\alpha\right)\left[(1+i_A)^2-1\right] - \phi\delta_0 + \phi\delta_1\xi_j - \left(\overline{\theta}-\alpha\right)(1-\pi)\, i_L^h\right]}{2\omega\left(1-\overline{\theta}\right)+\phi\delta_1^2} \tag{27}$$

We can also write $\frac{\partial \Upsilon_k}{\partial i_L^h} = 0$ as:

$$\begin{aligned} i_L^h \;=\; & \frac{\left[\frac{\psi}{1-\pi} + \frac{\alpha\left[\alpha\omega+\delta_1\left(1-\theta_h+\overline{\theta}-\alpha\right)\right]}{2\left(1-\overline{\theta}\right)}\right]\left[(1+i_A)^2-1\right]}{2\psi + \frac{\alpha(1-\pi)}{2\left(1-\overline{\theta}\right)}\left[\alpha\omega+\delta_1\left(\overline{\theta}-\alpha\right)\right]} \\ & + \frac{(1-\delta_0)\left(\overline{\theta}-\alpha\right) - \frac{\alpha\phi\delta_0\delta_1}{2\left(1-\overline{\theta}\right)} + \alpha\omega\xi_j - \delta_1\left[\overline{\theta}-\alpha+\frac{\alpha\phi\delta_1}{2\left(1-\overline{\theta}\right)}\right]\left(\xi_k-\xi_j\right)}{2\psi + \frac{\alpha(1-\pi)}{2\left(1-\overline{\theta}\right)}\left[\alpha\omega+\delta_1\left(\overline{\theta}-\alpha\right)\right]} \end{aligned} \tag{28}$$

Remark 1 *As long as the big bank's inequality constraints are non-binding, the equilibrium is a triple* $\left\{\xi_j, \xi_k, i_L^h\right\}$ *that solves (26), (27), and (28). We must therefore check that the solution to these equations satisfies* $\xi_k \geq 0$ *along with* $R_k > \alpha X_k$ *and* $\mu_j > 0$*. We also need to check* $W_j \leq X_j$ *and* $W_k \leq X_k$ *so that deposits are non-negative. Finally, we want to check that* $i_L^\ell = 0$ *does not result in a liquidity shortage when the big bank realizes* θ_ℓ *at* $t=1$*.*

The rest of this proof focuses on $\alpha = 0$. Notice $\xi_j = 0$ from (26). As discussed in the main text, we also want $\xi_k = 0$. Subbing $\alpha = 0$ and $\xi_j = \xi_k = 0$ into (27) and (28) yields:

$$\delta_1\left[\frac{\left(1-\theta_h+\overline{\theta}\right)\left[(1+i_A)^2-1\right]-\phi\delta_0}{\overline{\theta}(1-\pi)} - i_L^h\right] = 0 \tag{29}$$

$$i_L^h = \frac{(1+i_A)^2-1}{2(1-\pi)} + \frac{\overline{\theta}(1-\delta_0)}{2\psi} \tag{30}$$

To verify $\xi_k = 0$, we must verify that (29) holds when i_L^h is given by (30). This requires either $\delta_1 = 0$ or:

$$\phi = \frac{1}{\delta_0}\left[1 - \theta_h + \frac{\overline{\theta}}{2}\right]\left[(1+i_A)^2 - 1\right] - \frac{\overline{\theta}^2 (1-\pi)(1-\delta_0)}{2\psi\delta_0} \equiv \phi^* \tag{31}$$

In other words, we can use either $\delta_1 = 0$ or the combination of $\delta_1 > 0$ and $\phi = \phi^*$ to get ξ_k exactly zero at $\alpha = 0$. Note that $W_j \leq X_j$ and $W_k \leq X_k$ are trivially true with $\xi_j = \xi_k = 0$. We now need to check $R_k > \alpha X_k$ and $\mu_j > 0$. Using (14) and (30), rewrite $\mu_j > 0$ as:

$$\frac{(1+i_A)^2 - 1}{1-\pi} > \frac{\overline{\theta}(1-\delta_0)}{\psi} \tag{32}$$

Note that condition (32) is also sufficient for $\phi^* > 0$. With $\mu_j > 0$ verified, we can substitute $\alpha = 0$ into equation (22) to get $R_j = 0$. The next step is to check $R_k > \alpha X_k$ which is simply $R_k > 0$ at $\alpha = 0$. Recall that R_k is given by equation (25). Use $\alpha = 0$ and $\xi_j = \xi_k = 0$ along with i_L^h as per (30) to rewrite equation (25) as:

$$R_k = \theta_h \delta_0 + \frac{\overline{\theta}(1-\delta_0)}{2} - \psi \frac{(1+i_A)^2 - 1}{2(1-\pi)} \tag{33}$$

The condition for $R_k > 0$ is therefore:

$$\frac{(1+i_A)^2 - 1}{1-\pi} < \frac{\overline{\theta}(1-\delta_0)}{\psi} + \frac{2\delta_0\theta_h}{\psi} \tag{34}$$

The last step is to check that there is sufficient liquidity at $t = 1$ when the big bank's liquidity shock is low. The demand for liquidity in this case will be $\overline{\theta} X_j + \theta_\ell X_k$. The supply of liquidity will be $R_j + R_k$ since we have fixed $i_L^\ell = 0$. We already know $\xi_j = \xi_k = 0$ at $\alpha = 0$. Therefore, $X_j = 1 - \delta_0$ and $X_k = \delta_0$. We also know $R_j = 0$ and R_k as per (33). Therefore, $R_j + R_k \geq \overline{\theta} X_j + \theta_\ell X_k$ can be rewritten as:

$$\frac{(1+i_A)^2 - 1}{1-\pi} \leq \frac{2\delta_0(\theta_h - \theta_\ell)}{\psi} - \frac{\overline{\theta}(1-\delta_0)}{\psi} \tag{35}$$

Condition (35) is stricter than (34) so we can drop (34). We now just need to make sure that conditions (32) and (35) are not mutually exclusive. Using $\overline{\theta} \equiv \pi\theta_\ell + (1-\pi)\theta_h$, this requires:

$$\theta_\ell < \left[1 - \frac{1-\delta_0}{\delta_0 + \pi(1-\delta_0)}\right]\theta_h \tag{36}$$

The right-hand side of (36) is positive if and only if:

$$\pi > \frac{1-2\delta_0}{1-\delta_0} \tag{37}$$

Therefore, with θ_ℓ sufficiently low and π sufficiently high, conditions (32) and (35) define a non-empty interval for i_A, completing the proof. ■

Proof of Proposition 5

Fixed Funding Share Impose $\alpha = \overline{\theta}$ and $\delta_1 = 0$ on equations (26), (27), and (28). The resulting system can be written as $\xi_k = 0$ and:

$$\xi_j = \frac{\frac{\overline{\theta}\psi}{2}\left[(1+i_A)^2 - 1\right]}{2\psi\left(1-\overline{\theta}\right) + \omega\overline{\theta}^2(1-\pi)} \tag{38}$$

$$i_L^h = \frac{\left[\frac{\psi(1-\overline{\theta})}{1-\pi} + \omega\overline{\theta}^2\right]\left[(1+i_A)^2 - 1\right]}{2\psi\left(1-\overline{\theta}\right) + \omega\overline{\theta}^2(1-\pi)} \tag{39}$$

With $\delta_1 = 0$ in equations (23) and (24), the funding shares are $X_j = 1-\delta_0$ and $X_k = \delta_0$. Impose along with $\alpha = \overline{\theta}$ on equations (22) and (25) to get:

$$R_k = \theta_h\delta_0 + \omega\overline{\theta}\xi_j - \psi i_L^h$$

$$R_j + R_k = \overline{\theta}(1-\delta_0) + \theta_h\delta_0 - \psi i_L^h$$

where ξ_j and i_L^h are given by (38) and (39) respectively. We now need to go through all the steps in Remark 1 to establish the equilibrium for $\alpha = \overline{\theta}$ and fixed funding shares. Using equations (14) and (39), we can see that $\mu_j > 0$ is trivially true. Using $\xi_k = 0$ and $X_k = \delta_0$, we can also see that $W_k \leq X_k$ is trivially true. The condition for $W_j \leq X_j$ is:

$$\frac{(1+i_A)^2 - 1}{1-\pi} \leq \frac{2(1-\delta_0)}{\psi}\left[\overline{\theta} + \frac{2\psi\left(1-\overline{\theta}\right)}{\omega\overline{\theta}(1-\pi)}\right] \tag{40}$$

The conditions for $R_k > \overline{\theta}X_k$ and $R_j + R_k \geq \overline{\theta}X_j + \theta_\ell X_k$ are respectively:

$$\frac{(1+i_A)^2 - 1}{1-\pi} < \frac{2\pi(\theta_h - \theta_\ell)\delta_0}{\psi} \tag{41}$$

$$\frac{(1+i_A)^2-1}{1-\pi} \leq \frac{2\psi\left(1-\overline{\theta}\right)+\omega\overline{\theta}^2(1-\pi)}{\psi\left(1-\overline{\theta}\right)+\omega\overline{\theta}^2(1-\pi)} \frac{(\theta_h-\theta_\ell)\,\delta_0}{\psi} \tag{42}$$

Now, for the interbank rate to increase when moving from $\alpha = 0$ to $\alpha = \overline{\theta}$, we need (39) to exceed (30). Equivalently, we need:

$$\frac{(1+i_A)^2-1}{1-\pi} > \frac{\overline{\theta}\,(1-\delta_0)}{\psi}\left[1+\frac{2\psi\left(1-\overline{\theta}\right)}{\omega\overline{\theta}^2(1-\pi)}\right] \tag{43}$$

We must now collect all the conditions involved in the $\alpha = 0$ and $\alpha = \overline{\theta}$ equilibria and make sure they are mutually consistent. There are two lowerbounds on i_A, namely (32) and (43). Condition (43) is clearly stricter so it is the relevant lowerbound. There are also four upperbounds on i_A, namely (35), (40), (41), and (42). For the lowerbound in (43) to not violate any of these upperbounds, we need:

$$\frac{\psi\left(1-\overline{\theta}\right)}{\omega(1-\pi)} < \overline{\theta}^2 \min\left\{\frac{\pi\,(\theta_h-\theta_\ell)\,\delta_0}{\overline{\theta}\,(1-\delta_0)}-\frac{1}{2}, \frac{(\theta_h-\theta_\ell)\,\delta_0}{\overline{\theta}\,(1-\delta_0)}-1\right\}$$

This inequality is only possible if the right-hand side is positive. Therefore, we need:

$$\theta_\ell < \left[1-\frac{1-\delta_0}{\min\left\{\delta_0+\pi\,(1-\delta_0)\,,\pi\,(1+\delta_0)\right\}}\right]\theta_h \tag{44}$$

Once again, the right-hand side must be positive so we need:

$$\pi > \max\left\{\frac{1-2\delta_0}{1-\delta_0}, \frac{1-\delta_0}{1+\delta_0}\right\} \tag{45}$$

Notice that (44) and (45) are just refinements of (36) and (37). We can now conclude that the model with fixed funding shares generates the desired results under the following conditions: π sufficiently high, θ_ℓ and $\frac{\psi}{\omega}$ sufficiently low, and i_A within an intermediate range. $\square$

Endogenous Funding Share Return to equations (26), (27), and (28). Impose $\alpha = \overline{\theta}$ and $\delta_1 = \omega$ with $\phi = \phi^*$ as per (31). Combine to get:

$$i_L^h = \frac{(1+i_A)^2-1}{1-\pi} - \frac{\left[\frac{2\psi}{1-\pi}+\frac{\omega\overline{\theta}^2}{2\left(1-\overline{\theta}\right)+\phi^*\omega}\right]\frac{(1+i_A)^2-1}{2(1-\pi)} - \frac{\omega\overline{\theta}^3(1-\delta_0)}{2\psi\left[2\left(1-\overline{\theta}\right)+\phi^*\omega\right]}}{\frac{2\psi}{1-\pi}+\frac{\omega\overline{\theta}^2}{2\left(1-\overline{\theta}\right)}\left[2+\frac{\phi^*\omega}{2\left(1-\overline{\theta}\right)+\phi^*\omega}\right]} \tag{46}$$

$$\xi_k = \frac{\frac{\overline{\theta}(1-\pi)}{2}\left[\frac{\overline{\theta}(1-\delta_0)}{\psi} + \left(\frac{\phi^*\omega}{1-\overline{\theta}} - 1\right)\frac{(1+i_A)^2-1}{1-\pi} - \frac{\phi^*\omega}{1-\overline{\theta}} i_L^h\right]}{2\left(1-\overline{\theta}\right) + \phi^*\omega} \tag{47}$$

We now need to go through the steps in Remark 1 to establish the equilibrium for $\alpha = \overline{\theta}$ and endogenous funding shares. The expressions here are more complicated so we proceed by finding one value of i_A that satisfies all the steps in Remark 1. A continuity argument will then allow us to conclude that all the steps are satisfied for a non-empty range of i_A.

Consider i_A such that:

$$\frac{(1+i_A)^2 - 1}{1-\pi} = \frac{\overline{\theta}}{\psi} \tag{48}$$

Substituting into (31) then pins down ϕ^* as:

$$\phi^* = \frac{\overline{\theta}\,(1-\pi)}{\psi}\left[\frac{1-\theta_h}{\delta_0} + \frac{\overline{\theta}}{2}\right] \tag{49}$$

From the proof of Proposition 4, we already have (32) and (35) as restrictions on i_A. We also have (36) as an upperbound on θ_ℓ and (37) as a lowerbound on π. It is easy to see that i_A as defined in (48) satisfies (32). For (48) to also satisfy (35), we need:

$$\theta_\ell < \left[1 - \frac{2-\delta_0}{2\delta_0 + \pi\,(2-\delta_0)}\right]\theta_h \tag{50}$$

$$\pi > \frac{2-3\delta_0}{2-\delta_0} \tag{51}$$

Conditions (50) and (51) are stricter than (36) and (37). We can thus drop (36) and (37).

The first step is to verify $\mu_j > 0$. Use (14) and (46) to write $\mu_j > 0$ as:

$$\frac{(1+i_A)^2 - 1}{1-\pi}\left[1 + \frac{2\psi\left[2\left(1-\overline{\theta}\right) + \phi^*\omega\right]}{\omega\overline{\theta}^2\,(1-\pi)}\right] > \frac{\overline{\theta}\,(1-\delta_0)}{\psi}$$

This is true by condition (32).

The second step is to verify $\xi_k > 0$. Substituting (46) into (47), we see that we need:

$$\frac{(1+i_A)^2 - 1}{1-\pi}\left[1 - \frac{\phi^*}{\frac{2\left(1-\overline{\theta}\right)}{\omega} + \frac{\overline{\theta}^2(1-\pi)}{\psi}}\right] < \frac{\overline{\theta}\,(1-\delta_0)}{\psi} \tag{52}$$

Using i_A as per (48) and ϕ^* as per (49):

$$\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)} < \frac{\overline{\theta}}{2\delta_0^2}\underbrace{\left[1-\theta_h-\overline{\theta}\delta_0\left(\delta_0-\frac{1}{2}\right)\right]}_{\text{call this } Z_1} \tag{53}$$

If $Z_1 > 0$, then (53) requires $\frac{\psi}{\omega}$ sufficiently low. Note that $Z_1 > 0$ can be made true for any $\delta_0 \in (0,1)$ by assuming $\overline{\theta} < 2\left(1-\theta_h\right)$ or, equivalently, $\theta_\ell < \frac{2-(3-\pi)\theta_h}{\pi}$. This is another positive ceiling on θ_ℓ provided $\pi > 3 - \frac{2}{\theta_h}$.

The third step is to verify $R_k > \overline{\theta}X_k$. Use $\alpha = \overline{\theta}$ and $\delta_1 = \omega$ to rewrite (24) and (25) as:

$$X_k = \delta_0 + \omega\left(\xi_k - \xi_j\right) \tag{54}$$

$$R_k = \delta_0\theta_h + \omega\theta_h\xi_k - \omega\left(\theta_h - \overline{\theta}\right)\xi_j - \psi i_L^h \tag{55}$$

Therefore, $R_k > \overline{\theta}X_k$ requires:

$$i_L^h < \frac{\delta_0\left(\theta_h-\overline{\theta}\right)}{\psi} + \frac{\omega\left(\theta_h-\overline{\theta}\right)}{\psi}\left(\xi_k-\xi_j\right) + \frac{\omega\overline{\theta}}{\psi}\xi_j$$

Use (47) to replace ξ_k and (26) with $\alpha = \overline{\theta}$ to replace ξ_j:

$$\begin{aligned} &\left[1+\frac{\omega\overline{\theta}\left(1-\pi\right)}{2\psi\left(1-\overline{\theta}\right)}\left[\overline{\theta}-\frac{2\left(1-\overline{\theta}\right)\left(\theta_h-\overline{\theta}\right)}{2\left(1-\overline{\theta}\right)+\phi^*\omega}\right]\right]i_L^h \\ < \;& \frac{\theta_h-\overline{\theta}}{\psi}\left[\delta_0+\frac{\omega\overline{\theta}^2\left(1-\pi\right)\left(1-\delta_0\right)}{2\psi\left[2\left(1-\overline{\theta}\right)+\phi^*\omega\right]}\right] - \frac{\omega\overline{\theta}\left[\left(1+i_A\right)^2-1\right]}{2\psi}\left[\frac{3\left(\theta_h-\overline{\theta}\right)}{2\left(1-\overline{\theta}\right)+\phi^*\omega}-\frac{\overline{\theta}}{1-\overline{\theta}}\right]\end{aligned}$$

Now use (46) to replace i_L^h and rearrange to isolate i_A:

$$\begin{aligned} &\frac{\left(1+i_A\right)^2-1}{1-\pi}\left[2\theta_h-\frac{3\overline{\theta}}{2}+\frac{2\psi\left(1-\overline{\theta}\right)}{\omega\overline{\theta}\left(1-\pi\right)}+\frac{\omega\overline{\theta}^2\left(1-\pi\right)}{4\psi\left(1-\overline{\theta}\right)}\left(3\theta_h-4\overline{\theta}\right)+\frac{\phi^*}{\overline{\theta}}\left[\frac{\omega\overline{\theta}^2}{1-\overline{\theta}}+\frac{\psi}{1-\pi}\right]\right] \\ < \;& \left[\frac{2\delta_0\left(\theta_h-\overline{\theta}\right)}{1-\pi}\frac{2\left(1-\overline{\theta}\right)+\phi^*\omega}{\omega\overline{\theta}}-\frac{\overline{\theta}^2\left(1-\delta_0\right)}{2\psi}\right]\left[1+\frac{\overline{\theta}^2}{2}\frac{\omega\left(1-\pi\right)}{\psi\left(1-\overline{\theta}\right)}\right] \\ &+\left(\theta_h-\overline{\theta}\right)\frac{\overline{\theta}}{\psi}\left[\frac{\omega\phi^*\delta_0}{2\left(1-\overline{\theta}\right)}+\left(1-\delta_0\right)\left[1+\frac{3\overline{\theta}^2}{4}\frac{\omega\left(1-\pi\right)}{\psi\left(1-\overline{\theta}\right)}\right]\right]\end{aligned}$$

We can simplify a bit further by using (31) to replace all instances of $\phi^*\delta_0$ then grouping like terms:

$$\frac{(1+i_A)^2-1}{1-\pi}\left[\begin{array}{l}\theta_h-\frac{\overline{\theta}}{2}+\frac{2\psi\left(1-\overline{\theta}\right)}{\omega\overline{\theta}(1-\pi)}-\frac{\omega\overline{\theta}^3(1-\pi)}{4\psi\left(1-\overline{\theta}\right)}+\frac{\phi^*}{\overline{\theta}}\left[\frac{\omega\overline{\theta}^2}{1-\overline{\theta}}+\frac{\psi}{1-\pi}\right]\\ -\left(\theta_h-\overline{\theta}\right)(1-\theta_h)\left[\frac{2}{\overline{\theta}}+\frac{3\overline{\theta}}{2}\frac{\omega(1-\pi)}{\psi\left(1-\overline{\theta}\right)}\right]\end{array}\right]$$
$$<\quad\left[\frac{4\delta_0\left(1-\overline{\theta}\right)\left(\theta_h-\overline{\theta}\right)}{\omega\overline{\theta}\,(1-\pi)}-\frac{\overline{\theta}^2\,(1-\delta_0)}{2\psi}\right]\left[1+\frac{\overline{\theta}^2}{2}\frac{\omega\,(1-\pi)}{\psi\left(1-\overline{\theta}\right)}\right]$$

Substitute i_A as per (48) and ϕ^* as per (49) then rearrange:

$$\frac{\psi\left(1-\overline{\theta}\right)}{\omega\,(1-\pi)}\left[\theta_h-\frac{\overline{\theta}\,(1+\delta_0)}{2}+\left[\overline{\theta}+\frac{1-\theta_h}{\delta_0}+\frac{2}{\overline{\theta}}\frac{\psi\left(1-\overline{\theta}\right)}{\omega\,(1-\pi)}\right]\left[1-\frac{2\delta_0\left(\theta_h-\overline{\theta}\right)}{\overline{\theta}}\right]\right] \tag{56}$$
$$<\quad\underbrace{\frac{\overline{\theta}}{\delta_0}\left[\frac{\overline{\theta}^2\delta_0^2}{4}+\frac{\delta_0}{2}\left[3\left(\theta_h-\overline{\theta}\right)(1-\theta_h)-\overline{\theta}^2\right]-\overline{\theta}\,(1-\theta_h)\right]}_{\text{call this } Z_2}$$

Condition (56) will be true for $\frac{\psi}{\omega}$ sufficiently low if $Z_2>0$. Use $\overline{\theta}\equiv\pi\theta_\ell+(1-\pi)\,\theta_h$ to rewrite $Z_2>0$ as:

$$\pi^2\,(\theta_h-\theta_\ell)^2-2\left[\theta_h+\frac{(2+3\delta_0)\,(1-\theta_h)}{\delta_0\,(2-\delta_0)}\right]\pi\,(\theta_h-\theta_\ell)+\theta_h\left[\theta_h+\frac{4\,(1-\theta_h)}{\delta_0\,(2-\delta_0)}\right]<0$$

Based on the roots of this quadratic, we can conclude that $Z_2>0$ requires:

$$\pi\,(\theta_h-\theta_\ell)>\theta_h+\frac{(2+3\delta_0)\,(1-\theta_h)}{\delta_0\,(2-\delta_0)}-\sqrt{\frac{1-\theta_h}{2-\delta_0}\left(6\theta_h+\frac{(2+3\delta_0)^2\,(1-\theta_h)}{\delta_0^2\,(2-\delta_0)}\right)} \tag{57}$$

Condition (57) is satisfied by $\theta_\ell=0$ and $\pi=1$. The left-hand side is decreasing in θ_ℓ and increasing in π so it follows that $Z_2>0$ requires θ_ℓ sufficiently low and π sufficiently high.

The fourth step is to verify $W_j\leq X_j$. Use $W_j=\omega\xi_j$ and (23) with $\delta_1=\omega$ to rewrite $W_j\leq X_j$ as:

$$\xi_k\leq\frac{1-\delta_0}{\omega}$$

Now use (47) with i_L^h as per (46) to replace ξ_k. Substitute i_A as per (48) and ϕ^* as per (49).

Rearrange to isolate all terms with $\frac{\psi\left(1-\overline{\theta}\right)}{\omega(1-\pi)}$ on one side. The condition for $W_j \leq X_j$ becomes:

$$\begin{aligned} & \frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}\left[\frac{\overline{\theta}^2}{2}+\left(1-\delta_0\right)\left[\overline{\theta}^2+\frac{\overline{\theta}\left(1-\theta_h\right)}{\delta_0}+2\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}\right]\right] \\ \geq\ & \underbrace{\frac{\overline{\theta}^3}{4}\left[\left(1-\theta_h\right)\left(3-\frac{2}{\delta_0}\right)-\overline{\theta}\left(1-\frac{\delta_0}{2}\right)\right]}_{\text{call this } Z_3} \end{aligned} \tag{58}$$

If $Z_3 < 0$, then we can get $W_j \leq X_j$ without requiring a floor on $\frac{\psi}{\omega}$. This is useful since our other steps required $\frac{\psi}{\omega}$ sufficiently low. A sufficient condition for $Z_3 < 0$ is $\delta_0 \leq \frac{2}{3}$.

The fifth step is to verify $W_k \leq X_k$. Use $W_k = \omega\xi_k$ and (54) to rewrite $W_k \leq X_k$ as:

$$\xi_j \leq \frac{\delta_0}{\omega}$$

Now use (26) with $\alpha = \overline{\theta}$ and i_L^h as per (46) to replace ξ_j. Substitute i_A as per (48) and ϕ^* as per (49). Rearrange to isolate all terms with $\frac{\psi\left(1-\overline{\theta}\right)}{\omega(1-\pi)}$ on one side. The condition for $W_k \leq X_k$ becomes:

$$\begin{aligned} & \frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}\left[1-3\delta_0-\frac{2\delta_0}{\overline{\theta}}\left[\frac{1-\theta_h}{\delta_0}+\frac{2}{\overline{\theta}}\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}\right]\right] \\ \leq\ & \underbrace{\frac{\overline{\theta}}{2}\left[\left(1-\theta_h\right)\left(3-\frac{1}{\delta_0}\right)-\overline{\theta}\left(\frac{1}{2}-\delta_0\right)\right]}_{\text{call this } Z_4} \end{aligned} \tag{59}$$

Condition (59) will be true for $\frac{\psi}{\omega}$ sufficiently low if $Z_4 > 0$. Use the definition of $\overline{\theta}$ to rewrite $Z_4 > 0$ as:

$$\pi\left(\theta_h-\theta_\ell\right)\delta_0\left(1-2\delta_0\right) > \theta_h\delta_0\left(1-2\delta_0\right)-2\left(1-\theta_h\right)\left(3\delta_0-1\right) \tag{60}$$

If $\delta_0 \geq \frac{1}{2}$, then (60) is always true. If $\delta_0 < \frac{1}{2}$, then (60) reduces to:

$$\theta_\ell < \frac{1}{\pi}\left[\frac{2\left(1-\theta_h\right)\left(3\delta_0-1\right)}{\delta_0\left(1-2\delta_0\right)}-\theta_h\left(1-\pi\right)\right]$$

This is a positive ceiling on θ_ℓ provided $\pi > 1-\frac{2(1-\theta_h)(3\delta_0-1)}{\theta_h\delta_0(1-2\delta_0)}$ with $\delta_0 > \frac{1}{3}$. Therefore, (60) is guaranteed by θ_ℓ sufficiently low, π sufficiently high, and $\delta_0 > \frac{1}{3}$.

The sixth step is to verify feasibility of $i_L^\ell = 0$. This requires $R_j + R_k \geq \overline{\theta}X_j + \theta_\ell X_k$. Use

(22) with $\alpha = \overline{\theta}$ to replace R_j. The desired inequality becomes:

$$R_k \geq \theta_\ell X_k + \omega\overline{\theta}\xi_j$$

Substituting X_k and R_k as per equations (54) and (55):

$$i_L^h \leq \frac{\theta_h - \theta_\ell}{\psi}\left[\delta_0 + \omega\left(\xi_k - \xi_j\right)\right]$$

Use (47) to replace ξ_k. Also use (26) with $\alpha = \overline{\theta}$ to replace ξ_j. Rearrange to isolate i_L^h then use (46) to replace i_L^h. Substitute i_A as per (48) and ϕ^* as per (49). Rearrange to isolate all terms with $\frac{\psi(1-\overline{\theta})}{\omega(1-\pi)}$ on one side. The feasibility condition for $i_L^\ell = 0$ becomes:

$$\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}\left[\begin{array}{c} \frac{\overline{\theta}(5-\delta_0)}{4} - \left(\theta_h - \theta_\ell\right)\left[\frac{1-\theta_h}{\overline{\theta}} + \frac{2\delta_0 - 1}{2}\right] \\ + \frac{1-\theta_h}{2\delta_0} + \left[1 - \frac{2(\theta_h - \theta_\ell)\delta_0}{\overline{\theta}}\right]\frac{1}{\overline{\theta}}\frac{\psi\left(1-\overline{\theta}\right)}{\omega(1-\pi)} \end{array}\right] \leq \frac{3\overline{\theta}}{4}\underbrace{\left[\left(1-\theta_h\right)\left[\theta_h - \theta_\ell - \frac{\overline{\theta}}{\delta_0}\right] - \frac{\overline{\theta}^2}{2}\right]}_{\text{call this } Z_5} \quad (61)$$

Condition (61) will be true for $\frac{\psi}{\omega}$ sufficiently low if $Z_5 > 0$. Use the definition of $\overline{\theta}$ to rewrite $Z_5 > 0$ as:

$$\pi^2\left(\theta_h - \theta_\ell\right)^2 - 2\left[\pi\theta_h + \frac{\left(\pi + \delta_0\right)\left(1-\theta_h\right)}{\delta_0}\right]\left(\theta_h - \theta_\ell\right) + \theta_h\left[\theta_h + \frac{2\left(1-\theta_h\right)}{\delta_0}\right] < 0$$

Based on the roots of this quadratic, we can conclude that $Z_5 > 0$ requires:

$$\theta_\ell < \frac{1}{\pi^2}\left[\sqrt{2\pi\theta_h\left(1-\theta_h\right) + \frac{\left(\pi+\delta_0\right)^2\left(1-\theta_h\right)^2}{\delta_0^2}} - \frac{\left(\pi+\delta_0\right)\left(1-\theta_h\right)}{\delta_0} - \theta_h\pi\left(1-\pi\right)\right]$$

This is a positive upperbound on θ_ℓ provided $\frac{\theta_h(1-\pi)^2}{2(1-\theta_h)} + \frac{1-\pi}{\delta_0} < 1$. Therefore, $Z_5 > 0$ requires θ_ℓ sufficiently low and π sufficiently high.

It now remains to check that the interbank rate increases when moving from $\alpha = 0$ to $\alpha = \overline{\theta}$. This requires (46) to exceed (30) or, equivalently:

$$\frac{\left(1+i_A\right)^2 - 1}{1-\pi} > \left(1-\delta_0\right)\left[\frac{\overline{\theta}}{\psi} + \frac{4\left(1-\overline{\theta}\right)}{\omega\overline{\theta}\left(1-\pi\right)}\frac{2\left(1-\overline{\theta}\right) + \phi^*\omega}{2\left(1-\overline{\theta}\right) + 3\phi^*\omega}\right]$$

Using i_A as per (48) and ϕ^* as per (49):

$$\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}\left[\frac{1-\theta_h}{\delta_0}+\frac{\overline{\theta}\left(1-2\delta_0\right)}{2\left(1-\delta_0\right)}+\frac{2}{\overline{\theta}}\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}\right]<\frac{3\overline{\theta}^2}{4\left(1-\delta_0\right)}\left[1-\theta_h+\frac{\overline{\theta}\delta_0}{2}\right] \tag{62}$$

The right-hand side is positive so (62) will be true for $\frac{\psi}{\omega}$ sufficiently low.

Putting everything together, we have shown that the model with endogenous funding shares generates the desired results under the following conditions: π sufficiently high, θ_ℓ and $\frac{\psi}{\omega}$ sufficiently low, $\delta_0 \in \left(\frac{1}{3}, \frac{2}{3}\right)$, and i_A as per (48). The results then extend to a non-empty range of i_A by continuity. □

Comparison We now compare the interbank rate increases in the fixed share and endogenous share models. Notice from the proof of Proposition 4 that the interbank rate at $\alpha = 0$ is the same in both models. Therefore, we just need to show that the interbank rate in the endogenous share model exceeds the interbank rate in the fixed share model at $\alpha = \overline{\theta}$. In other words, we need to show that (46) exceeds (39) for a given set of parameters. This reduces to:

$$\frac{\left(1+i_A\right)^2-1}{1-\pi}\left[1-\frac{\phi^*}{\frac{2\left(1-\overline{\theta}\right)}{\omega}+\frac{\overline{\theta}^2\left(1-\pi\right)}{\psi}}\right]<\frac{\overline{\theta}\left(1-\delta_0\right)}{\psi}$$

which is exactly (52), where (52) was the condition for $\xi_k > 0$ at $\alpha = \overline{\theta}$ in the endogenous share model. To complete the proof, we must now show that there are indeed parameters that satisfy the conditions in both models. For $\alpha = 0$, we imposed conditions (32) and (35) along with π sufficiently high and θ_ℓ sufficiently low. These conditions applied to both models. For $\alpha = \overline{\theta}$ in the fixed share model, we also imposed conditions (40), (41), (42), and (43) along with $\frac{\psi}{\omega}$ sufficiently low. For $\alpha = \overline{\theta}$ in the endogenous share model, we added $\delta_0 \in \left(\frac{1}{3}, \frac{2}{3}\right)$ and i_A in the neighborhood of (48). In (50) and (51), we showed that π sufficiently high and θ_ℓ sufficiently low make (48) satisfy condition (35). We have also shown that condition (43) is stricter than condition (32). Therefore, we just need to show that (48) satisfies conditions (40), (41), (42), and (43). Substituting i_A as per (48) into these conditions produces the following inequalities which we must check:

$$\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}>\frac{\overline{\theta}^2\left(2\delta_0-1\right)}{4\left(1-\delta_0\right)} \tag{63}$$

$$\theta_\ell<\left[1-\frac{1}{\pi\left(1+2\delta_0\right)}\right]\theta_h \tag{64}$$

$$\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}\left[1-\frac{2\left(\theta_h-\theta_\ell\right)\delta_0}{\overline{\theta}}\right]<\overline{\theta}^2\left[\frac{\left(\theta_h-\theta_\ell\right)\delta_0}{\overline{\theta}}-1\right] \tag{65}$$

$$\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}<\frac{\overline{\theta}^2\delta_0}{2\left(1-\delta_0\right)} \tag{66}$$

A sufficient condition for (63) is $\delta_0 \leq \frac{1}{2}$ which is still consistent with $\delta_0 \in \left(\frac{1}{3}, \frac{2}{3}\right)$. Condition (64) is just another positive upperbound on θ_ℓ provided $\pi > \frac{1}{1+2\delta_0}$. In other words, (64) is satisfied by θ_ℓ sufficiently low and π sufficiently high. Condition (65) will be true for $\frac{\psi}{\omega}$ sufficiently low if $\left(\theta_h-\theta_\ell\right)\delta_0 > \overline{\theta}$ or, equivalently, $\theta_\ell < \left[1-\frac{1}{\delta_0+\pi}\right]\theta_h$ with $\pi > 1-\delta_0$ which again means θ_ℓ sufficiently low and π sufficiently high. Finally, condition (66) is clearly satisfied by $\frac{\psi}{\omega}$ sufficiently low. □ ■

Proof of Proposition 6

Evaluate (26) at $\alpha = \overline{\theta}$ then subtract (47) to get:

$$\xi_j-\xi_k \overset{sign}{=} \left[\frac{\left(1+i_A\right)^2-1}{1-\pi}-\frac{\overline{\theta}\left(1-\delta_0\right)}{\psi}\right]+2\left[\frac{\left(1+i_A\right)^2-1}{1-\pi}-i_L^h\right]$$

The expression in the first set of square brackets is positive by condition (32). The expression in the second set of square brackets is proportional to μ_j. The proof of Proposition 5 established $\mu_j > 0$. Therefore, $\xi_j > \xi_k$ at $\alpha = \overline{\theta}$.

Now consider total credit:

$$TC \equiv 1-R_j-R_k$$

Use market clearing as per (12) to replace $R_j + R_k$:

$$TC = 1-\overline{\theta}X_j-\theta_h X_k+\psi i_L^h$$

Use (23) and (24) to replace X_j and X_k:

$$TC = 1-\overline{\theta}-\left(\theta_h-\overline{\theta}\right)\delta_0+\delta_1\left(\theta_h-\overline{\theta}\right)\left(\xi_j-\xi_k\right)+\psi i_L^h$$

Proposition 5 showed $i_L^h\big|_{\alpha=\overline{\theta}} > i_L^h\big|_{\alpha=0}$. We also know $\xi_j = \xi_k = 0$ at $\alpha = 0$ and $\xi_j > \xi_k$ at $\alpha = \overline{\theta}$. Therefore, we can conclude $TC|_{\alpha=\overline{\theta}} > TC|_{\alpha=0}$.

Finally, we want to show that the loan-to-deposit ratios of big and small banks converge. The equilibrium has $\tau_j = 1$, meaning that small banks move all WMPs (and the associated investments) off-balance-sheet. The loan-to-deposit ratio of the representative small bank is then $\lambda_j \equiv 1 - \frac{R_j}{X_j - W_j}$. The equilibrium also has $\tau_k = 0$, meaning that the big bank records everything on-balance-sheet. Its loan-to-deposit ratio is then $\lambda_k \equiv 1 - \frac{R_k}{X_k}$. Proposition 4 established $R_k > 0 = R_j$ at $\alpha = 0$ so it follows that $\lambda_k|_{\alpha=0} < 1 = \lambda_j|_{\alpha=0}$. To show convergence, we just need to show $\lambda_k|_{\alpha=\overline{\theta}} > \lambda_k|_{\alpha=0}$ since $\lambda_j|_{\alpha=\overline{\theta}} < \lambda_j|_{\alpha=0}$ follows immediately from equation (22). Use $X_j + X_k = 1$ along with the definition of λ_k to rewrite (12) as:

$$\psi i_L^h = \overline{\theta} + \left[\theta_h - \overline{\theta} - (1 - \lambda_k)\right] X_k - R_j$$

We know $i_L^h\big|_{\alpha=\overline{\theta}} > i_L^h\big|_{\alpha=0}$ so it must be the case that:

$$\left[\theta_h - \overline{\theta} - (1 - \lambda_k|_{\alpha=\overline{\theta}})\right] X_k|_{\alpha=\overline{\theta}} - R_j|_{\alpha=\overline{\theta}} > \left[\theta_h - \overline{\theta} - (1 - \lambda_k|_{\alpha=0})\right] X_k|_{\alpha=0}$$

Proposition 4 also established $\xi_j = \xi_k = 0$ at $\alpha = 0$. Substituting into equation (24) then implies $X_k = \delta_0$ at $\alpha = 0$ so:

$$\lambda_k|_{\alpha=\overline{\theta}} \frac{X_k|_{\alpha=\overline{\theta}}}{\delta_0} - \lambda_k|_{\alpha=0} > \underbrace{\frac{R_j|_{\alpha=\overline{\theta}}}{\delta_0} - \left[1 - \pi\left(\theta_h - \theta_\ell\right)\right]\left[1 - \frac{X_k|_{\alpha=\overline{\theta}}}{\delta_0}\right]}_{\text{call this } Z_6}$$

We have shown $\xi_j > \xi_k$ at $\alpha = \overline{\theta}$ so equation (24) also implies $\frac{X_k|_{\alpha=\overline{\theta}}}{\delta_0} \leq 1$ for any $\delta_1 \geq 0$. Therefore, $Z_6 \geq 0$ will be sufficient for $\lambda_k|_{\alpha=\overline{\theta}} > \lambda_k|_{\alpha=0}$. If $\delta_1 = 0$, then $Z_6 \propto R_j|_{\alpha=\overline{\theta}} \geq 0$. If $\delta_1 = \omega$, then we can rewrite $Z_6 \geq 0$ as:

$$1 - \delta_0 - \omega\xi_k \geq \frac{1 - \pi\left(\theta_h - \theta_\ell\right)}{\overline{\theta}} \omega\left(\xi_j - \xi_k\right) \tag{67}$$

where ξ_j is given by (26) with $\alpha = \overline{\theta}$ and ξ_k is given by (47). Use these expressions to substitute out ξ_j and ξ_k then use equation (46) to substitute out i_L^h. Evaluate i_A at (48) and ϕ^* at (49) to rewrite (67) as:

$$\frac{4\psi\left(1 - \overline{\theta}\right)(1 - \delta_0)}{\omega\overline{\theta}\,(1 - \pi)} + \underbrace{\overline{\theta}\,(2 - 3\delta_0) + (1 - \theta_h)\left(\frac{2}{\delta_0} - 3 - \delta_0\right)}_{\text{call this } \Delta(\delta_0)}$$

$$\geq\ -\frac{\overline{\theta}^2}{4}\frac{\omega\,(1 - \pi)}{\psi\left(1 - \overline{\theta}\right)}\underbrace{\left[2\overline{\theta}\,(1 - 2\delta_0) + (1 - \theta_h)\left(\frac{4}{\delta_0} - 6 - 3\delta_0\right)\right]}_{\text{call this } \widetilde{\Delta}(\delta_0)}$$

A sufficient condition for this is $\min\left\{\Delta\left(\delta_0\right), \widetilde{\Delta}\left(\delta_0\right)\right\} \geq 0$. Notice $\Delta'\left(\cdot\right) < 0$ and $\widetilde{\Delta}'\left(\cdot\right) < 0$. Also notice $\min\left\{\Delta\left(\frac{1}{2}\right), \widetilde{\Delta}\left(\frac{1}{2}\right)\right\} > 0$ and $\min\left\{\Delta\left(\frac{2}{3}\right), \widetilde{\Delta}\left(\frac{2}{3}\right)\right\} < 0$. Therefore, there is a threshold $\overline{\delta_0} \in \left(\frac{1}{2}, \frac{2}{3}\right)$ such that $\delta_0 \leq \overline{\delta_0}$ guarantees $Z_6 \geq 0$. ■

Appendix B – Deposit and WMP Demands

Here we sketch a simple household maximization problem which generates the demands in equations (1) and (2). There is a continuum of ex ante identical households indexed by $i \in [0,1]$. Each household is endowed with X units of funding. Let D_{ij} and W_{ij} denote the deposits and WMPs purchased by household i from bank j, where:

$$\sum_j \left(D_{ij} + W_{ij}\right) \leq X \tag{68}$$

Assume that buying W_{ij} entails a transaction cost of $\frac{1}{2\omega_0}W_{ij}^2$, where $\omega_0 > 0$. As per the main text, the rate of return on the WMP is zero if withdrawn early and ξ_j otherwise. The rate of return on deposits is always zero and the average probability of early withdrawal is $\overline{\theta}$. The household requires subsistence consumption of X in each state, above which it is risk neutral. If the household were to bypass the banking system and invest in long-term projects directly, it would fall below subsistence in the state where it needs to liquidate early. Therefore, the household does not invest directly. Instead, it chooses D_{ij} and W_{ij} for each j to maximize:

$$\sum_j \left(D_{ij} + \left[1 + \left(1 - \overline{\theta}\right)\xi_j\right]W_{ij} - \frac{W_{ij}^2}{2\omega_0}\right)$$

subject to (68) holding with equality.[43] The first order condition with respect to W_{ij} is:

$$W_{ij} = \left(1 - \overline{\theta}\right)\omega_0\xi_j \tag{69}$$

Substituting (69) into (68) when the latter holds with equality gives the household's total deposit demand, $D_i \equiv \sum_j D_{ij}$. The household is indifferent about the allocation of D_i across

[43] Here is how to recover the two-point distribution of idiosyncratic bank shocks in Section 4.1 from the household withdrawals. Each household has probability θ_ℓ of being hit by an idiosyncratic consumption shock at $t = 1$ and having to withdraw all of its funding early. This results in each bank losing fraction θ_ℓ of its deposits and WMPs at $t = 1$. Then $\theta_h - \theta_\ell$ of the remaining $1 - \theta_\ell$ households observe a sunspot and withdraw all of their funding from $1 - \pi$ banks at $t = 1$. The $\theta_h - \theta_\ell$ households and $1 - \pi$ banks involved in the sunspot are chosen at random. Note $\overline{\theta} \equiv \pi\theta_\ell + (1 - \pi)\theta_h$.

banks so we assume that it simply allocates D_i uniformly. For J banks, this yields:

$$D_{ij} = \frac{X}{J} - \frac{\left(1-\overline{\theta}\right)\omega_0}{J}\xi_j - \frac{(J-1)\left(1-\overline{\theta}\right)\omega_0}{J}\frac{1}{J-1}\sum_{x\neq j}\xi_x \tag{70}$$

With a unit mass of ex ante identical households, $W_j = W_{ij}$ and $D_j = D_{ij}$. As J approaches a unit mass of equally-weighted banks, (69) and (70) belong to the family of functions specified by (1) and (2).

Appendix C – Benchmark with Aggregate Shock

Consider the benchmark model (only price-taking banks) in Sections 4.1 and 4.2 but with an aggregate interbank shock. In particular, the interbank rate is i_L^ℓ with probability π and i_L^h with probability $1-\pi$. The expected interbank rate is $i_L^e \equiv \pi i_L^\ell + (1-\pi)\, i_L^h$. We will specify how i_L^ℓ and i_L^h are determined shortly. In the meantime, banks take both as given.

The objective function of the representative bank simplifies to:

$$\Upsilon_j = (1+i_A)^2 (X_j - R_j) + (1+i_L^e) R_j - \left[X_j + i_L^e \overline{\theta} X_j + \left(1-\overline{\theta}\right)\xi_j W_j\right] - \tfrac{\phi}{2} X_j^2$$

This is identical to the benchmark model except with the expected interbank rate i_L^e instead of the deterministic i_L. Therefore, the first order conditions are still given by equations (7) to (9) but with i_L^e in place of i_L.

The goal is to show that i_L^e is always highest at $\alpha = 0$. The proof follows Proposition 3 but, to proceed, we must replace the deterministic market clearing condition (equation (4)) with conditions for each realization of the aggregate shock. We model the shock as a shock to the aggregate demand for liquidity at $t = 1$. In particular, aggregate liquidity demand is $\overline{\theta}X - \varepsilon$ with probability π and $\overline{\theta}X$ with probability $1-\pi$, where $\varepsilon > 0$. The interbank rates are then i_L^ℓ and i_L^h respectively. To avoid liquidity shortages, we need these rates to satisfy:

$$R_j + \psi i_L^\ell \geq \overline{\theta}X - \varepsilon \tag{71}$$

$$R_j + \psi i_L^h \geq \overline{\theta}X \tag{72}$$

The equilibrium i_L^h solves (72) with equality. If $i_L^h \leq \frac{\varepsilon}{\psi}$, then we can set $i_L^\ell = 0$. Otherwise, the equilibrium i_L^ℓ solves (71) with equality.

Let i^e_{L0} denote the expected interbank rate at $\alpha = 0$ and let $i^e_{L1}(\rho_2)$ denote the expected interbank rate at some $\alpha > 0$. Using (71) and (72), we can write:

$$i^e_{L1}(\rho_2) = \frac{\overline{\theta}X}{\psi} - \frac{R_{j1}(\rho_2)}{\psi} - \frac{\pi}{\psi}\min\left\{\overline{\theta}X - R_{j1}(\rho_2), \varepsilon\right\} \tag{73}$$

where $R_{j1}(\rho_2)$ is reserve holdings at the $\alpha > 0$ being considered. The proof of $i^e_{L1}(\rho_2) \leq i^e_{L0}$ proceeds by contradiction. In particular, suppose $i^e_{L1}(\rho_2) > i^e_{L0}$. Then (7) implies $\mu_j > 0$ at $\alpha = 0$. Complementary slackness then implies $R_j = 0$ at $\alpha = 0$ so we can write:

$$i^e_L = \frac{\overline{\theta}X}{\psi} - \frac{\pi}{\psi}\min\left\{\overline{\theta}X, \varepsilon\right\} \tag{74}$$

Subtract (74) from (73) to get:

$$i^e_{L1}(\rho_2) = i^e_{L0} - \frac{R_{j1}(\rho_2)}{\psi} + \frac{\pi}{\psi}\left[\min\left\{\overline{\theta}X, \varepsilon\right\} - \min\left\{\overline{\theta}X - R_{j1}(\rho_2), \varepsilon\right\}\right]$$

There are three cases. If $\varepsilon \leq \overline{\theta}X - R_{j1}(\rho_2)$, then:

$$i^e_{L1}(\rho_2) = i^e_{L0} - \frac{R_{j1}(\rho_2)}{\psi}$$

If $\overline{\theta}X - R_{j1}(\rho_2) < \varepsilon < \overline{\theta}X$, then:

$$i^e_{L1}(\rho_2) = i^e_{L0} - \frac{1-\pi}{\psi}R_{j1}(\rho_2) - \frac{\pi}{\psi}\left(\overline{\theta}X - \varepsilon\right)$$

If $\overline{\theta}X \leq \varepsilon$, then:

$$i^e_{L1}(\rho_2) = i^e_{L0} - \frac{1-\pi}{\psi}R_{j1}(\rho_2)$$

In each case, $i^e_{L1}(\rho_2) > i^e_{L0}$ would require $R_{j1}(\rho_2) < 0$ which is impossible. ■

Appendix D – Money Multiplier Calculation

A simple money multiplier calculation will help assess the contribution of China's RMB 4 trillion fiscal stimulus package to (i) more aggressive lending by the Big Four and (ii) China's aggregate credit boom.

The size of the stimulus is S and the fraction to be financed by the Big Four is q. To finance qS, the Big Four make a one-time transfer of qS from excess reserves to loans.

We will treat the Big Four as a closed system, meaning that their loans do not end up in deposit accounts at the SMBs. With a currency ratio of c and a reserve ratio of r, the multiplier process increases loans and deposits at the Big Four by $\frac{qS}{1-(1-r)(1-c)}$ and $\frac{(1-c)qS}{1-(1-r)(1-c)}$ respectively. We assume a conservative currency ratio ($c = 0.05$) so as not to understate the effect of the stimulus package on Big Four loans. Since our goal is to get an indication of the effects of the stimulus package had nothing else changed, we set $r = 0.35$, recalling from Figure 6 that the loan-to-deposit ratio of big banks averaged just over 0.6 between 2005 and 2008.

For the fraction of stimulus financed by the Big Four, we use $q = 0.65$. The Big Four had a market share (as measured by deposits) of roughly 55% in 2007. They may have been willing to pitch in a bit more but there is little to suggest they were expected to finance a disproportionate share of the stimulus package. The details of the package were such that the central government could only fund up to 33% of the planned investment/expenditure. The rest of the funds were to be borrowed by local governments. In modern-day China, the central government exercises only indirect control over the Big Four (e.g., through top personnel decisions) and can exercise exactly the same control over the joint-stock banks since they also operate nationally. Local governments have no administrative control over commercial banks. Their only leverage is economic (e.g., where to put local government savings) and it tends to work better with smaller/locally-operating banks such as city banks. In short, local governments are simply not powerful enough to force the Big Four (or even the joint-stock banks) to lend huge amounts of money that they otherwise would not have lent.

The results of our calculation suggest that China's stimulus package can account for up to RMB 6.8 trillion of new loans and up to RMB 6.5 trillion of new deposits at the Big Four since the end of 2008. *Taking these amounts out*, loans and deposits at the Big Four would have grown at annualized rates of 12.9% and 9.8% respectively from 2008 to 2014. The Big Four's loan-to-deposit ratio would have then increased from 0.57 in 2008 to 0.67 in 2014. This counterfactual estimate of what would have happened to the Big Four's loan-to-deposit ratio absent stimulus is similar to what actually happened with the stimulus (an increase from 0.57 in 2008 to 0.70 in 2014 as per Figure 6).

We now extend the calculation to estimate how much of the overall increase in China's credit-to-savings ratio can be reasonably explained by stimulus alone. Suppose the remaining $(1-q)\,S$ of stimulus was financed by SMBs. This amount will also go through the multiplier process, except with a lower reserve ratio (call it $\widetilde{r}$) since SMBs have higher loan-to-deposit ratios than the Big Four. We set $\widetilde{r} = 0.15$ based on the average balance data for 2007 in Figure 6. Combining the calculations for the SMBs with the calculations for the Big Four, we find that the stimulus package explains around 40% of the 10 percentage point increase in China's credit-to-savings ratio since 2007.