9 Online Appendix

In this online Appendix, we first define the recursive competitive equilibrium. We then provide proof of various lemmas and propositions in Section 2. We also provide the details of constructing empirical variables using COMPUSTAT (North America) Annual Data and those for VAR analysis.

9.1 Recursive Competitive Equilibrium

Let k and K be the individual and aggregate capital, respectively.

Definition 1 *A recursive competitive equilibrium with a constant aggregate technology Z for the simple economy consists of a capital allocation rule for the type-c projects, $k^c : R^+ \times R^+ \rightarrow R^+$, $k^c = k^c(K;Z)$, a value function for the projects to the lender, $V : R^+ \times R^+ \rightarrow R^+$, $V = V(K;Z)$, a capital allocation rule for the type-u projects, $k^u : R^+ \times R^+ \rightarrow R^+$, $k^u = k^u(K;Z)$, a saving decision rule for the household, $\gamma : R^+ \times R^+ \times R^+ \rightarrow R^+$, $k' = \gamma(k, K; Z)$, an interest rate function, $r : R^+ \times R^+ \rightarrow R^+$, $r = r(K;Z)$, and a law of motion of aggregate capital, $\Gamma : R^+ \times R^+ \rightarrow R^+$, $K' = \Gamma(K;Z)$, such that*

1. $k^c(K;Z)$ solves

$$k^c(K;Z) = \arg\max_{k^c} ZF(k^c) - (r(K;Z) + \delta) k^c, \tag{38}$$

   subject to

$$D(k^c) \leq \phi\beta V(\Gamma(K;Z);Z), \tag{39}$$

   where $V(K;Z)$ satisfies

$$V(K;Z) = \max_{k} ZF(k) - (r(K;Z) + \delta) k + \phi\beta V(\Gamma(K;Z);Z). \tag{40}$$

2. $k^u(K;Z)$ satisfies

$$ZF'(k^u(K;Z)) = r(K;Z) + \delta. \tag{41}$$

3. $\gamma\left(k,K;Z\right)$ solves

$$\gamma\left(k,K;Z\right)=\arg\max_{k'}u\left(\left(1+r(K;Z)\right)k-k'\right)+\beta v\left(k',\Gamma\left(K;Z\right);Z\right), \tag{42}$$

where

$$v\left(k,K;Z\right)=u\left(\left(1+r(K;Z)\right)k-\gamma\left(k,K;Z\right)\right)+\beta v\left(\gamma\left(k,K;Z\right),\Gamma\left(K;Z\right);Z\right).$$

4. $\Gamma\left(K;Z\right)$ is consistent with $\gamma\left(k,K;Z\right)$:

$$\Gamma\left(K;Z\right)=\gamma\left(K,K;Z\right). \tag{43}$$

5. The capital market clears:

$$K=\left(1-\eta\right)k^{u}\left(K;Z\right)+\eta k^{c}\left(K;Z\right). \tag{44}$$

When $\eta=0$ - i.e., no projects require working capital - the recursive equilibrium reduces to the one in the standard neo-classical growth model. The equilibrium can be fully characterized by solving a fixed-point of $\Gamma\left(K;Z\right)$, the law of motion of the aggregate capital. When $\eta>0$, the recursive equilibrium entails an additional fixed-point of $V\left(K;Z\right)$. Moreover, $V\left(K;Z\right)$ and $\Gamma\left(K;Z\right)$ affect each other. On the one hand, $\Gamma\left(K;Z\right)$ affects $V\left(K;Z\right)$ through the future project value, $V\left(\Gamma\left(K;Z\right);Z\right)$. On the other hand, $V\left(\Gamma\left(K;Z\right);Z\right)$ determines capital allocation, which, in turn, pins down the interest rate. The chain builds up a channel through which $V\left(K;Z\right)$ influences the interest rate and, thus, the aggregate saving decision. Lemma 2 below shows explicitly how $V\left(K;Z\right)$ and $\Gamma\left(K;Z\right)$ interact with each other.

9.2 Proof of Lemma 1

Suppose that the financial constraint is binding in the steady state. Then, by (39), we have $k^{c*} = (\phi\beta V^*)^{\frac{1}{\alpha}}$, where

$$V^* = \frac{\pi^*}{1-\phi\beta} = \frac{(1-\alpha)\, Z \left(\frac{\alpha Z}{1/\beta - 1 + \delta}\right)^{\frac{\alpha}{1-\alpha}}}{1-\phi\beta} \tag{45}$$

Clearly, $V^* > 0$. k^{c*} follows immediately from (39) and (45).

$$k^{c*} = \left[\frac{\phi\beta\,(1-\alpha)\, Z}{1-\phi\beta}\right]^{\frac{1}{\alpha}} \left(\frac{\alpha Z}{1/\beta - 1 + \delta}\right)^{\frac{1}{1-\alpha}}. \tag{46}$$

The household Euler equation implies that $r^* = 1/\beta - 1$. (41) shows that $Z\alpha\,(k^{u*})^{\alpha-1} = r^* + \delta$, which solves $k^{u*} = \left(\frac{\alpha Z}{1/\beta - 1 + \delta}\right)^{\frac{1}{1-\alpha}}$. Since

$$\frac{k^{c*}}{k^{u*}} = \left[\frac{\phi\beta\,(1-\alpha)\, Z}{1-\phi\beta}\right]^{\frac{1}{\alpha}}, \tag{47}$$

Condition (7) ensures that $k^{c*} < k^{u*}$; i.e., the financial constraint is indeed binding in the steady state.

9.3 Lemma 2

The following lemma shows how $V(K;Z)$ and $\Gamma(K;Z)$ interact with each other in the simple model.

Lemma 2 *If the financial constraint is always binding for the type-c projects, the recursive equilibrium can be characterized by* $V(K;Z)$ *and* $\Gamma(K;Z)$*, which solve*

$$V(K;Z) = (1-\alpha)\, ZF\left(\frac{K - \eta D^{-1}\left(\phi\beta V\left(\Gamma(K;Z);Z\right)\right)}{1-\eta}\right) + \phi\beta V\left(\Gamma(K;Z);Z\right), \tag{48}$$

and

$$\Gamma(K;Z) = \arg\max_{K'} u\left(f(K;Z) - K'\right) + \beta V^h\left(K';Z\right), \tag{49}$$

where

$$f(K;Z) = (1 + r(K;Z))\, K, \tag{50}$$

$$V^h(K;Z) = u(f(K;Z) - \Gamma(K;Z)) + \beta V^h(\Gamma(K;Z);Z).$$

(48) is derived from (40). Specifically, the choice of k in (40) follows a similar first order condition as (41), since neither the lender nor the type-u entrepreneur is financially constrained. Accordingly, $k(K;Z) = k^u(K;Z)$. Given $F(\cdot) = (\cdot)^\alpha$ and $r(K;Z)$ in (41), the period profit for the lender in (40) becomes $(1-\alpha) ZF(k(K,Z))$. In addition, $k(K;Z) = k^u(K;Z) = \frac{K - \eta k^c(K;Z)}{1-\eta} = \frac{K - \eta D^{-1}(\phi\beta V(\Gamma(K;Z);Z))}{1-\eta}$, where the second and third equalities derive from (44) and (39), respectively. Therefore, we obtain (48).

A combination of (42) and (43) leads to (49). For analytical convenience, we define $f(K;Z)$ in (50) as the household's wealth after production takes place: i.e., the sum of her net-of-depreciation capital $(1-\delta)K$ and her capital income $r(K;Z)K$. The system of nonlinear functional equations (48) and (49) suggests that characterizing the recursive equilibrium with $\eta > 0$ be much harder. Yet, some important local properties can be established by parameterizing $F(\cdot)$, $D(\cdot)$ and $u(\cdot)$ in a fairly standard way.

9.4 Proof of Proposition 1

Since the financial constraint for the type-c projects is binding in the steady state, the continuity of f, Γ and V established below guarantees a neighborhood of K^* where the constraint is always binding. The rest of the proof entails three steps.

1. For any $f(K;Z)$, prove that $\Gamma(K;Z) = \beta f(K;Z)$.

2. For any $f(K;Z)$, prove that $V(K;Z)$ is unique and satisfies $V_K(K;Z) > 0$ and $V(K;Z_2) > V(K;Z_1), \forall Z_2 > Z_1$.

3. Prove that the economy contains a stable steady state, that is, $\beta f_K(K^*;Z) < 1$.

9.4.1 Step 1: Proof of $\Gamma(K;Z) = \beta f(K;Z)$

We characterize the equilibrium by Lemma 2. The representative household's Euler equation can be written as

$$\frac{f(\Gamma(K;Z);Z) - \Gamma(\Gamma(K;Z);Z)}{f(K;Z) - \Gamma(K;Z)} = \beta \frac{f(\Gamma(K;Z);Z)}{\Gamma(K;Z)}.$$

Clearly, $\Gamma(K;Z) = \beta f(K;Z)$ is a solution to the Euler equation. Note that the fixed-point of (49) is identical to that in the standard growth model. Moreover, for any differentiable f with $f_K(K;Z) > 0$ and $f_{KK}(K;Z) < 0$, we can directly apply the standard recursive method in Stokey and Lucas (1989) to prove that $\Gamma(K;Z) = \beta f(K;Z)$ is a unique solution.

9.4.2 Step 2: Proof of various features of $V(K;Z)$

We have established that for any $f \in S^f$, $\Gamma(K;Z) = \beta f(K;Z)$. In this step, we prove that for any $f \in S^f$, there is a unique $V(K;Z)$. Moreover, $V_K(K;Z) > 0$ and $V(K;Z_2) > V(K;Z_1), \forall Z_2 > Z_1$.

Since $F(K) = D(K) = K^\alpha$ and $D^{-1}(\phi\beta V(\Gamma(K;Z);Z)) = (\phi\beta V(\Gamma(K;Z);Z))^{1/\alpha}$, (48) defines the following operator T:

$$(TV)(K;Z) = (1-\alpha) ZF\left(\frac{K - \eta D^{-1}(\phi\beta V(\Gamma(K;Z);Z))}{1-\eta}\right) + \phi\beta V(\Gamma(K;Z);Z). \tag{51}$$

Since Γ is differentiable, it is straightforward that T maps the set of differentiable functions to itself.

We next show that T is a contraction mapping by applying Blackwell's sufficient conditions: i.e., monotonicity and discounting. To prove monotonicity, we differentiate the RHS of (51) with respect to $V(\Gamma(K;Z);Z)$. The derivative, denoted by T_V, is

$$T_V = \phi\beta - (1-\alpha) Z\phi\beta\frac{\eta}{1-\eta}\left(\frac{K - \eta(\phi\beta V(\Gamma(K;Z);Z))^{1/\alpha}}{1-\eta}\right)^{\alpha-1}\left(\phi\beta V^h(\Gamma(K;Z);Z)\right)^{1/\alpha-1}.$$

Monotonicity can be proved if T_V is positive.[38] Notice that

$$\frac{K - \eta(\phi\beta V(\Gamma(K;Z);Z))^{1/\alpha}}{1-\eta} = k^u(K;Z) > k^c(K;Z) = (\phi\beta V(\Gamma(K;Z);Z))^{1/\alpha}.$$

[38] A positive T_V implies that for any x, $y \in S^V$, $x \geq y$ implies $T(x) \geq T(y)$.

Therefore, when the financial constraint is binding, we can show that

$$\begin{aligned} T_V &> \phi\beta - (1-\alpha)\, Z\phi\beta\frac{\eta}{1-\eta}\left(\phi\beta V\left(\Gamma\left(K;Z\right);Z\right)\right)^{1-1/\alpha}\left(\phi\beta V\left(\Gamma\left(K;Z\right);Z\right)\right)^{1/\alpha-1} \\ &= \phi\beta - (1-\alpha)\, Z\phi\beta\frac{\eta}{1-\eta}. \end{aligned}$$

Moreover, $\phi\beta - (1-\alpha)\, Z\phi\beta\frac{\eta}{1-\eta} \geq \phi\beta - (1-\alpha)\, Z\phi\beta \geq 1 - \phi\beta - (1-\alpha)\, Z\phi\beta > 0$. The first inequality comes from the assumption $\eta \leq 1/2$, where the second inequality obtains under the assumption $\phi\beta \geq 1/2$, the last inequality obtains from (7). This proves monotonicity.

To prove discounting, we need

$$(T\left(V+a\right))\left(K;Z\right) \leq (T\left(V\right))\left(K;Z\right) + \phi\beta a,$$

where a is a positive real number. (51) gives

$$\begin{aligned} & (T\left(V+a\right))\left(K;Z\right) \\ =\ & (1-\alpha)\, Z\left(\frac{K-\eta\left(\phi\beta(V\left(\Gamma\left(K;Z\right);Z\right)+a)\right)^{\frac{1}{\alpha}}}{1-\eta}\right)^{\alpha} + \phi\beta V\left(\Gamma\left(K;Z\right);Z\right) + \phi\beta a. \end{aligned}$$

Since $\left(\frac{K-\eta(\phi\beta(V(\Gamma(K;Z);Z)+a))^{1/\alpha}}{1-\eta}\right)^{\alpha} < \left(\frac{K-\eta(\phi\beta V(\Gamma(K;Z);Z))^{1/\alpha}}{1-\eta}\right)^{\alpha}$ we have

$$\begin{aligned} (T\left(V+a\right))\left(K;Z\right) &< (1-\alpha)\, Z\left(\frac{K-\eta\left(\phi\beta V\left(\Gamma\left(K;Z\right);Z\right)\right)^{\frac{1}{\alpha}}}{1-\eta}\right)^{\alpha} + \phi\beta V\left(\Gamma\left(K;Z\right);Z\right) + \phi\beta a \\ &= (TV)\left(K;Z\right) + \phi\beta a. \end{aligned}$$

This proves discounting. Therefore, T satisfies both of Blackwell's sufficient conditions. It follows that T is a contraction and $V\left(K;Z\right) = (TV)\left(K;Z\right)$ has a unique fixed point.

Now, we derive $V_K\left(K;Z\right) > 0$. From (51) and (50), $V\left(K;Z\right)$ is the solution to

$$V\left(K;Z\right) = (1-\alpha)\, Z\left(\frac{K-\eta\left(\phi\beta V\left(\beta f\left(K;Z\right);Z\right)\right)^{\frac{1}{\alpha}}}{1-\eta}\right)^{\alpha} + \phi\beta V\left(\beta f\left(K;Z\right);Z\right). \quad (52)$$

Differentiating (52) with respect to K yields

$$\begin{aligned} V_K(K;Z) &= \alpha(1-\alpha) Z \left(\frac{K - \eta\left(\phi\beta V\left(\beta f(K;Z);Z\right)\right)^{\frac{1}{\alpha}}}{1-\eta} \right)^{\alpha-1} \\ &\quad \times \left[\frac{1 - \eta/\alpha\left(\phi\beta V\left(\beta f(K;Z);Z\right)\right)^{\frac{1}{\alpha}-1} \phi\beta V_K\left(\beta f(K;Z);Z\right)\beta f_K(K;Z)}{1-\eta} \right] \\ &\quad + \phi\beta V_K\left(\beta f(K;Z);Z\right)\beta f_K(K;Z). \end{aligned}$$

Now, we compute the derivative around the steady state. For notational convenience, we let X_K stand for $X_K(K^*;Z)$. Since $K^* = \Gamma(K^*;Z) = \beta f(K^*;Z)$ and $\alpha Z\left(k^{u*}\right)^{\alpha-1} = r^* + \delta = 1/\beta - (1-\delta)$, at the steady state, we have

$$V_K = (1-\alpha)\left(1/\beta - (1-\delta)\right)\left[\frac{1 - \eta/\alpha\left(k^{c*}\right)^{1-\alpha}\phi\beta V_K \beta f_K}{1-\eta}\right] + \phi\beta V_K \beta f_K. \tag{53}$$

Here, we use the fact that $\left(k^{c*}\right)^{\alpha} = \phi\beta V\left(\beta f(K^*;Z);Z\right)$. Rearranging (53) leads to

$$V_K = \frac{(1-\alpha)(1/\beta - 1 + \delta)/(1-\eta)}{1 + \phi\beta^2 f_K\left(\frac{\eta(1-\alpha)}{\alpha(1-\eta)}(1/\beta - 1 + \delta)\left(k^{c*}\right)^{1-\alpha} - 1\right)}. \tag{54}$$

(46) implies that

$$\left(k^{c*}\right)^{1-\alpha} = \left[\frac{\phi\beta(1-\alpha)Z}{1-\phi\beta}\right]^{\frac{1-\alpha}{\alpha}} \frac{\alpha Z}{1/\beta - 1 + \delta}. \tag{55}$$

Substituting (55) back into (54) yields

$$V_K = \frac{(1-\alpha)(1/\beta - 1 + \delta)/(1-\eta)}{1 + \phi\beta^2 f_K\left(\frac{\eta Z(1-\alpha)}{1-\eta}\left[\frac{\phi\beta(1-\alpha)Z}{1-\phi\beta}\right]^{\frac{1-\alpha}{\alpha}} - 1\right)}. \tag{56}$$

The assumption $\eta \leq 1/2$, $\phi\beta \geq 1/2$ and (7) implies that $\frac{\eta Z(1-\alpha)}{1-\eta} \leq Z(1-\alpha) \leq \frac{\phi\beta(1-\alpha)Z}{1-\phi\beta} < 1$. Therefore,

$$-1 < \frac{\eta Z(1-\alpha)}{1-\eta}\left[\frac{\phi\beta(1-\alpha)Z}{1-\phi\beta}\right]^{\frac{1-\alpha}{\alpha}} - 1 < 0.$$

As a result, with $\phi\beta^2 f_K < \beta f_K < 1$ (which will be proved in Step 3), (56) implies that

$$V_K > (1-\alpha)(1/\beta - 1 + \delta)/(1-\eta) > 0. \tag{57}$$

This proves $V_K > 0$.

We now prove that $V(K;Z_2) > V(K;Z_1)$ for any $Z_2 > Z_1$. The proof entails two steps. The first step constructs a sequence of value functions generated by an operator defined in (58). The sequence starts with the original value function, $V(K;Z_1)$, and converges to the new one, $V(K;Z_2)$. The second step proves the sequence to be monotonically increasing.

The operator is defined as follows.

$$\left(\tilde{T}V\right)(K;Z_1) = (1-\alpha) Z_2 \left(\frac{K - \eta \left(\phi\beta V\left(\Gamma\left(K;Z_1\right);Z_1\right)\right)^{\frac{1}{\alpha}}}{1-\eta} \right)^{\alpha} + \phi\beta V\left(\Gamma\left(K;Z_1\right);Z_1\right). \tag{58}$$

The only difference between (51) with $Z = Z_1$ and (58) is that Z outside F is replaced with Z_2. Following exactly the same proof as above for TV, we can show $\tilde{T}V$ satisfies both monotonicity and discounting and, thus, is a contraction mapping, which implies $\lim_{n\to\infty}\left(\tilde{T}V\right)^n(K;Z_1) = V(K;Z_2)$.

Next, we show that $\left(\tilde{T}V\right)^n(K;Z_1)$ is monotonically increasing. We first establish that $\left(\tilde{T}V\right)(K;Z_1) > V(K;Z_1)$. (58) implies

$$\begin{aligned} \left(\tilde{T}V\right)(K;Z_1) &= (1-\alpha)(Z_2 - Z_1)\left(k^u(K;Z_1)\right)^{\alpha} + (TV)(K;Z_1) \\ &> V(K;Z_1). \end{aligned} \tag{59}$$

The first line uses the facts that $k^u(K;Z_1) = \frac{K-\eta(\phi\beta V(\Gamma(K;Z_1);Z_1))^{1/\alpha}}{1-\eta}$. The inequality comes from the facts that $(TV)(K;Z_1) = V(K;Z_1)$, $Z_2 > Z_1$ and $\alpha \in (0,1)$.

We then proceed by showing that $\left(\tilde{T}V\right)^2(K;Z_1) > \left(\tilde{T}V\right)(K;Z_1)$. Following the similar proof of the monotonicity of T, we can establish the monotonicity of $\tilde{T}$ in (58), which ensures $\left(\tilde{T}V\right)^2(K;Z_1) > \left(\tilde{T}V\right)(K;Z_1)$, since $\left(\tilde{T}V\right)(K;Z_1) > V(K;Z_1)$. We can, thus, show that $\left(\tilde{T}V\right)^n(K;Z_1) > \cdots > \left(\tilde{T}V\right)(K;Z_1) > V(K;Z_1)$, which proves $V(K;Z_2) > V(K;Z_1)$ as $V(K;Z_2) = \lim_{n\to\infty}\left(\tilde{T}V\right)^n(K;Z_1)$.

9.4.3 Step 3: Proof of $\beta f_K(K^*; Z) < 1$

Using $F(K) = D(K) = K^\alpha$ and (41), equation (50) becomes

$$\frac{f(K;Z)}{K} = \alpha Z \left(\frac{K - \eta \left(\phi \beta V \left(\Gamma(K;Z);Z\right)\right)^{1/\alpha}}{1-\eta} \right)^{\alpha-1} + (1-\delta). \tag{60}$$

Differentiating (60) yields

$$\frac{f_K(K;Z)K - f(K;Z)}{K^2} = (\alpha - 1) \frac{ZF'(k^u(K;Z))}{k^u(K;Z)} \frac{\partial k^u(K;Z)}{\partial K}. \tag{61}$$

In the steady state, $K^* = \Gamma = \beta f$, $\Gamma_K = \beta f_K$ and $\alpha Z (k^{u*})^{\alpha-1} = r^* + \delta = 1/\beta - 1 + \delta$. Hence (61) becomes

$$\begin{aligned} (\beta f_K - 1) \frac{k^{u*}}{\beta K^*} &= (\alpha-1) \frac{(1/\beta - 1 + \delta)}{1-\eta} \left[1 - \frac{\eta}{\alpha} (k^{c*})^{1-\alpha} \phi \beta V_K \beta f_K \right] \\ &= (\alpha-1) \frac{(1/\beta - 1 + \delta)}{1-\eta} \frac{1 - \phi\beta^2 f_K}{1 - \phi \beta^2 f_K \left[1 - \frac{\eta Z(1-\alpha)}{1-\eta} \left(\frac{\phi\beta(1-\alpha)Z}{1-\phi\beta} \right)^{\frac{1-\alpha}{\alpha}} \right]} \end{aligned} \tag{62}$$

where the second equality is obtained by using (56). With $\frac{k^{u*}}{K^*} = \frac{1}{1-\eta+\eta\left[\frac{\phi\beta(1-\alpha)Z}{1-\phi\beta}\right]^{\frac{1}{\alpha}}}$, equation(62) restricts $\beta f_K(K^*; Z)$ to be a fixed point. Therefore, to prove that the economy contains a stable steady state, it is sufficient to prove that the left-hand side ("LHS" hereafter) of (62) crosses the right-hand side ("RHS" hereafter) of (62) at $\beta f_K < 1$.

The sufficient condition for the LHS of (62) to cross the RHS of (62) at $\beta f_K < 1$ can be derived as follows. Note that $\text{LHS}|_{\beta f_K = 0} = -\frac{k^{u*}}{\beta K^*}$ and $\text{LHS}|_{\beta f_K = 1} = 0$. $\text{RHS}|_{\beta f_K = 0} = (\alpha - 1)\frac{(1/\beta - 1 + \delta)}{1-\eta}$ and $\text{RHS}|_{\beta f_K = \frac{1}{\phi\beta}} = 0$. Moreover, $\frac{\partial LHS}{\partial \beta f_K} > 0$ and $\frac{\partial^2 LHS}{\partial (\beta f_K)^2} = 0$. And it is easy to show $\frac{\partial RHS}{\partial \beta f_K} = (1-\alpha)\frac{(1/\beta-1+\delta)}{1-\eta} \frac{\beta\phi \frac{\eta Z(1-\alpha)}{1-\eta}\left(\frac{\phi\beta(1-\alpha)Z}{1-\phi\beta}\right)^{\frac{1-\alpha}{\alpha}}}{\left(1-\phi\beta^2 f_K\left[1-\frac{\eta Z(1-\alpha)}{1-\eta}\left(\frac{\phi\beta(1-\alpha)Z}{1-\phi\beta}\right)^{\frac{1-\alpha}{\alpha}}\right]\right)^2} > 0$ and $\frac{\partial^2 RHS}{\partial (\beta f_K)^2} > 0$ when $\beta f_K \in \left[0, \frac{1}{\phi\beta}\right]$. Hence the sufficient condition for LHS to cross RHS of

(62) at $\beta f_K < 1$ is RHS$|_{\beta f_K=0}$>LHS$|_{\beta f_K=0}$, that is

$$(\alpha-1)\frac{(1/\beta-1+\delta)}{1-\eta} > -\frac{k^{u*}}{\beta K^*} = -\frac{1/\beta}{1-\eta+\eta\left(\frac{\phi\beta(1-\alpha)Z}{1-\phi\beta}\right)^{\frac{1}{\alpha}}}.$$

With assumption (7), it is sufficient that

$$\frac{1-\alpha}{1-\eta}(1/\beta-1+\delta) < 1/\beta.$$

Obviously, with $\eta \leq 1/2$, the above inequality is easily satisfied with a value of β close to 1.[39] Hence, equation (62) contains a root $\beta f_K(K^*;Z) < 1$.

9.5 Data Sources

The data sources used in Section 5 included two groups: 1. annual data used to estimate the relative capital productivity of constrained to unconstrained firms and its cyclicality; 2. quarterly data used to identify news shocks and to explore its impact on the above measure of capital misallocation. To be consistent, both groups of data sample between 1975 and 2010.

9.5.1 Data for Estimating the Relative Capital Productivity of Constrained to Unconstrained Firms and its Cyclicality

COMPUSTAT. Our data for Section 6.1 in the main text are taken from COMPUSTAT and consist of annual data from 1975 to 2010. We follow Covas and Den Hann (2011) in filtering the sample. Specifically, our sample includes firms listed on the three U.S. exchanges, NYSE, AMEX and Nasdaq, with a non-foreign incorporation code. We exclude financial firms (SIC codes 6000-6999), utilities (SIC codes 4900-4949), and firms involved in major mergers (COMPUSTAT footnote code AB) from the whole sample. We also exclude firms with a negative or missing value for the book value of assets, and firm-year observations that violate the accounting identity by more than ten percent of the book value of assets. Finally, we eliminate the firms most affected by the accounting change in 1988, namely, GM, GE, Ford, and Chrysler.

[39] In fact, if $\alpha > \eta$, as in our calibration, any $\beta > 0$ can satisfy this condition.

Firm size is proxied by the book value of assets (AT), deflated by the Producer Price Index (PPI). Capital income is measured as operating income before depreciation (OIBDP). Capital stock is given by net Plant, Property & Equipment (PPENT), lagged by one year. Moreover, when we compute the SA index, firm age is proxied by the number of years since the firm's first year of observation in COMPUSTAT. Our sample is comprised of all firm-year observations with positive capital income and a non-missing value for capital stock. The sample is an unbalanced panel, which includes $77,750$ observations, for an average of 1944 observations per year.

For firms with a fiscal year ending in the months January through May, we shift the observation to align it better with the observation for the macroeconomic variables. For example, a year t observation for a firm with a fiscal year ending in May corresponds to the period from June of year $t-1$ to May of year t. This observation enters our sample in year $t-1$.

Output and Deflator. Real GDP in Section 6.1 is measured by real gross value added of nonfinancial corporate business in billions chained (2005) dollars from the National Income and Product Accounts (Table 1.14). The Producer Price Index is given by the Producer Price Index for the industrial commodities from the Bureau of Labor Statistics.

Finally, Table A.1 is presented as follows.

Table A.1. Cross-Classification of Constraint Types

Financial Constraint Criteria	SA Index		Firm Size	
	Constrained	Unconstrained	Constrained	Unconstrained
1. SA Index				
Constrained	23756		20288	3468
Unconstrained		71194	3448	67746
2. Firm Size				
Constrained	20228	3448	23736	
Unconstrained	3468	67746		71214

Note: The SA index is constructed by (33). Firm size refers to one-year lagged book assets. The numbers in the table stand for the numbers of COMPUSTAT firms in each of the cross-classified category.

9.5.2 Data for Estimating VAR

The Relative Capital Productivity. The relative capital productivity of constrained to unconstrained firms is estimated using quarterly COMPUSTAT data over the period 1975Q2 to 2010Q4 and following the same empirical strategy as Section 6.1.3.

Aggregate TFP. The aggregate TFP measure is taken from Fernald (2009)'s total factor productivity (TFP series), updated on John Fernald's webpage.

Stock Prices. The measure of stock prices is the log of per capita real S&P 500 index. The S&P 500 composite index is taken from Robert Shiller's website. The price deflator is the price index for gross value added in the non-farm business sector, taken from the Bureau of Economic Analysis (Table 1.3.4). The population is civilian non-institutional population age 16 above from the Bureau of Labor Statistics. The stock index is converted to a quarterly frequency by taking the average of monthly stock index over each quarter.

9.6 Variable Definition

Variable definitions: All the variables are constructed using COMPUSTAT (North America) Annual Data. All names in parentheses refer to the COMPUSTAT item name.

Kaplan-Zingales (1997) index =-1.002*Cash flow + 0.283*Q + 3.319*Debt - 39.368*Dividends - 1.315*Cash balance.

Cash flow = Income before extraordinary items (ib) + Depreciation and amortization (dp) / Total assets (at).

Tobin'Q = Market value of assets (Total assets (at)+ Market value of common equity (csho*prcc_f) - Common equity (ceq) - Deferred taxes (txdb)) / Total assets (at).

Debt = Total debt (Debt in current liabilities (dlc) + Long-term debt (dltt)) / (Total debt + Total stockholders' equity (teq)).

Dividends = (Common dividends (dvc) + Preferred dividends (dvp)) /Total assets (at).

Cash balance = Cash and short-term investments (ch) / Total assets (at).

Whited-Wu (2006) index = −0.091*Cash flow + 0.062*Dividend dummy + 0.021*Long-term debt - 0.044* Size + 0.102* Industry sales growth - 0.035* Sales growth.

Dividend dummy is an indicator that takes the value of one if the firm pays positive dividends.

Long-term debt = Long-term debt (dltt) / total assets (at).

Size = log of Total assets (at).

Sales growth = (Net sales (sale) – Lagged Net sales) / Lagged Net sales

Industry sales growth = Sample average of the Sales growth of all firms in a three-digit SIC industry.

Payout ratio = (Cash dividends (dvp+dvc) + Repurchases (prstkc)) /Income before extraordinary items (ib).

SA (Hadlock and Pierce, 2010) index = (0.737* SA Size) + (0.043*SA Size^2) - (0.040*Age).

SA Size = log of min {Total assets (at), $4.5 billion}.

Age = min {Firm Age, thirty-seven years}.

Bond history dummy is an indicator that takes the value of one if the firm has ever issued corporate bond during the sample period.

External Finance dependence = (Capital expenditures (capx) - cash flow from operations)

Cash flow from operation =funds from operations (fopt) + decreases in inventories + decreases in accounts receivable + increases in account payable. When fopt is missing, funds from operations are defined as the sum of the following variables: Income before extraordinary items (ibc), depreciation and amortization (dpc), deferred taxes (txdc), equity in net loss/earnings (esubc), sale of property, plant and equipment and investments-gain/loss (sppiv), and funds from operations-other (fopo).